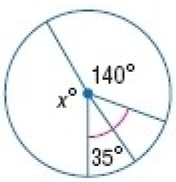
Find the value of *x*.

1. 

SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360.

$$\begin{aligned} 60+130+x &= 360 && \text{Sum of Central Angles} \\ 190+x &= 360 && \text{Simplify.} \\ x &= 360-190 && \text{Subtract 190 from each side.} \\ x &= 170 && \text{Simplify.} \end{aligned}$$

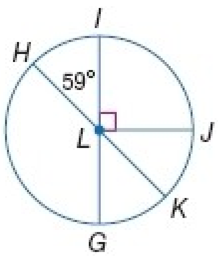
2. 

SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360.

$$\begin{aligned} 140+35+35+x &= 360 && \text{Sum of Central Angles} \\ 210+x &= 360 && \text{Simplify.} \\ x &= 360-210 && \text{Subtract 210 from each side.} \\ x &= 150 && \text{Simplify.} \end{aligned}$$

$\overline{HK}$ and $\overline{IG}$ are diameters of $\odot L$. Identify each arc as a *major arc*, *minor arc*, or *semicircle*. Then find its measure.



3. $m\widehat{IHJ}$

SOLUTION:

Here, $\widehat{IHJ}$ is the longest arc connecting the points *I* and *J* on $\odot L$. Therefore, it is a major arc.

$\widehat{IHJ}$ is a major arc that shares the same endpoints as minor arc *IJ*.

$$\begin{aligned} m(\text{arc}IHJ) &= 360 - m(\text{arc}IJ) && \text{Measure of Major Arc Rule} \\ &= 360 - 90 && \text{Substitution} \\ &= 270 && \text{Simplify.} \end{aligned}$$

4. $m\widehat{HI}$

SOLUTION:

Here, $\widehat{HI}$ is the shortest arc connecting the points *I* and *H* on $\odot L$. Therefore, it is a minor arc.

$$\begin{aligned} m(\text{arc}HI) &= m\angle HLI && \text{Measure of Minor Arc Rule} \\ &= 59 && \text{Substitution.} \end{aligned}$$

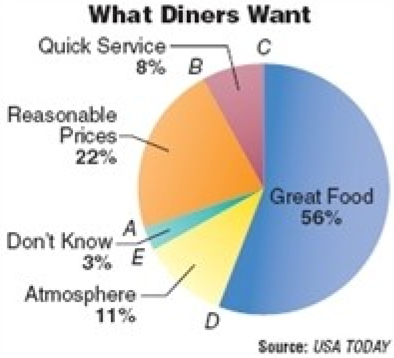
5. $m\widehat{HGK}$

SOLUTION:

Here, $\overline{HK}$ is a diameter. Therefore, $\widehat{HGK}$ is a semicircle.

The measure of a semicircle is 180, so $m\widehat{HGK} = 180.$

6. **RESTAURANTS** The graph shows the results of a survey taken by diners relating what is most important about the restaurants where they eat.



a. Find $m\widehat{AB}$.

b. Find $m\widehat{BC}$.

c. Describe the type of arc that the category Great Food represents.

SOLUTION:

a. Here, $\widehat{AB}$ is a minor arc.

The measure of the arc is equal to the measure of the central angle. Find the 22% of 360 to find the central angle.

$$\begin{aligned} m(\text{arc}\,AB) &= 0.22(360) && \text{Find 22\% of 360.} \\ &= 79.2 && \text{Simplify.} \end{aligned}$$

b. Here, $\widehat{BC}$ is a minor arc.

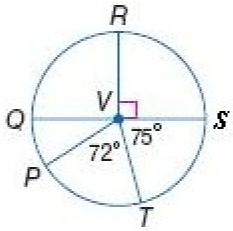
The measure of the arc is equal to the measure of the central angle. Find the 8% of 360 to find the central angle.

$$\begin{aligned} m(\text{arc}\,BC) &= 0.08(360) && \text{Find 8\% of 360.} \\ &= 28.8 && \text{Simplify.} \end{aligned}$$

c. The arc that represents the category Great Food $\widehat{CD}$, is the longest arc connecting the points *C* and *D*. Therefore, it is a major arc.

10-2 Measuring Angles and Arcs

$\overline{QS}$ is a diameter of $\odot V$. Find each measure.



7. $m\widehat{STP}$

SOLUTION:

$\widehat{ST}$ and $\widehat{SP}$ are adjacent arcs. The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$$\begin{aligned} m(\text{arc}STP) &= m(\text{arc}ST) + m(\text{arc}TP) && \text{Arc Addition Postulate} \\ &= 75 + 72 && m(\text{arc}ST) = m\angle SVT,\ m(arc\,TP) = m\angle TVP \\ &= 147 && \text{Simplify.} \end{aligned}$$

8. $m\widehat{QRT}$

SOLUTION:

$\widehat{QRS}$ and $\widehat{ST}$ are adjacent arcs. The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. Since $\overline{QS}$ is a diameter, arc *QRS* is a semicircle and has a measure of 180.

$$\begin{aligned} m(\text{arc}QRT) &= m(\text{arc}QRS) + m(\text{arc}ST) && \text{Arc Addition Postulate} \\ &= 180 + 75 && m(\text{arc}ST) = m\angle SVT \\ &= 255 && \text{Simplify.} \end{aligned}$$

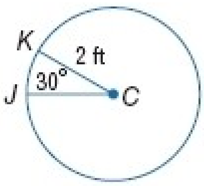
9. $m\widehat{PQR}$

SOLUTION:

If a set of adjacent arcs form a circle, then the sum of their measures is equal to 360. Since $\angle RVS$ is a right angle, $m\angle RVS = 90$.

$$\begin{aligned} m(\text{arc}PQR) + m(\text{arc}RS) + m(\text{arc}ST) + m(\text{arc}TP) &= 360 && \text{Arc Addition Postulate} \\ m(\text{arc}PQR) + 90 + 75 + 72 &= 360 && \text{Measure of arc equals measure of central angle.} \\ m(\text{arc}PQR) + 237 &= 360 && \text{Simplify.} \\ m(\text{arc}PQR) &= 123 && \text{Subtract 237 from each side.} \end{aligned}$$

Find the length of $\widehat{JK}$. Round to the nearest hundredth.

10. 

SOLUTION:

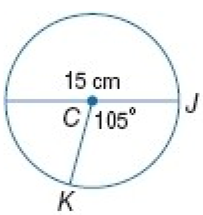
Use the arc length equation with $r = KC$ or 2 and $x = m\widehat{JK}$ or 30.

$$\ell = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation}$$

$$= \frac{30}{360} \cdot 2\pi(2) \quad \text{Substitution}$$

$$\approx 1.05 \quad \text{Use a calculator.}$$

Therefore, the length of $\widehat{JK}$ is about 1.05 feet.

11. 

SOLUTION:

The diameter of $\odot C$ is 15 centimeters, so the radius is 7.5 centimeters. The $m\widehat{JK} = m\angle KCJ$ or 105. Use the equation to find the arc length.

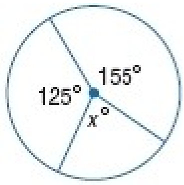
$$\ell = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation}$$

$$= \frac{105}{360} \cdot 2\pi(7.5) \quad \text{Substitution}$$

$$\approx 13.74 \quad \text{Use a calculator.}$$

Therefore, the length of $\widehat{JK}$ is about 13.74 centimeters.

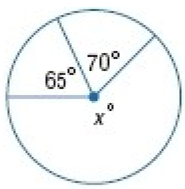
Find the value of *x*.

12. 

SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360.

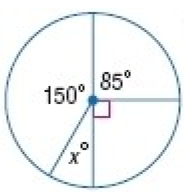
$$\begin{aligned} 125+155+x &= 360 && \text{Sum of Central Angles} \\ 280+x &= 360 && \text{Simplify.} \\ x &= 360-280 && \text{Subtract 280 from each side.} \\ x &= 80 && \text{Simplify.} \end{aligned}$$

13. 

SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360.

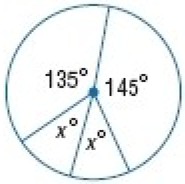
$$\begin{aligned} 65+70+x &= 360 && \text{Sum of Central Angles} \\ 135+x &= 360 && \text{Simplify.} \\ x &= 360-135 && \text{Subtract 135 from each side.} \\ x &= 225 && \text{Simplify.} \end{aligned}$$

14. 

SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360.

$$\begin{aligned} 150+85+90+x &= 360 && \text{Sum of Central Angles} \\ 325+x &= 360 && \text{Simplify.} \\ x &= 360-325 && \text{Subtract 325 from each side.} \\ x &= 35 && \text{Simplify.} \end{aligned}$$

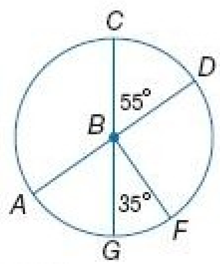
15. 

SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360.

$$\begin{aligned} 135+145+x+x &= 360 && \text{Sum of Central Angles} \\ 280+2x &= 360 && \text{Simplify.} \\ 2x &= 80 && \text{Subtract 280 from each side.} \\ x &= 40 && \text{Divide each side by 2.} \end{aligned}$$

$\overline{AD}$ and $\overline{CG}$ are diameters of $\odot B$. Identify each arc as a *major arc, minor arc,* or *semicircle*. Then find its measure.



16. $m\widehat{CD}$

SOLUTION:

Here, $\widehat{CD}$ is the shortest arc connecting the points *C* and *D* on $\odot B$. Therefore, it is a minor arc.

$$\begin{aligned} m(\text{arc } CD) &= m\angle CBD && \text{Measure of Minor Arc Rule} \\ &= 55 && \text{Substitution.} \end{aligned}$$

17. $m\widehat{AC}$

SOLUTION:

Here, $\widehat{AC}$ is the shortest arc connecting the points *A* and *C* on $\odot B$. Therefore, it is a minor arc.

Since $\overline{AD}$ is a diameter, arc *ACD* is a semicircle and has a measure of 180. Use angle addition to find the measure of arc *AC*.

$$\begin{aligned} m(\text{arc } ACD) &= m(\text{arc } AC) + m(\text{arc } CD) && \text{Arc Addition Postulate} \\ 180 &= m(\text{arc } AC) + 55 && m(\text{arc } CD) = m\angle CBD \\ 125 &= m(\text{arc } AC) && \text{Subtract 55 from each side.} \end{aligned}$$

Therefore, the measure of arc *AC* is 125.

10-2 Measuring Angles and Arcs

18. m(arc CFG)

SOLUTION:

Here, $\overline{CG}$ is a diameter. Therefore, arc CFG is a semicircle and m(arc CFG) = 180.

19. $m\widehat{CGD}$

SOLUTION:

Here, $\widehat{CGD}$ is the longest arc connecting the points C and D on $\odot B.$ Therefore, it is a major arc.
Arc CGD is a major arc that shares the same endpoints as minor arc CD.

$$\begin{aligned} m(\text{arc}\,CGD) &= 360 - m(\text{arc}\,CD) && \text{Measure of Major Arc Rule} \\ &= 360 - 55 && m(\text{arc}\,CD) = m\angle CBD \\ &= 270 && \text{Simplify.} \end{aligned}$$

Therefore, the measure of arc CGD is 305.

20. $m\widehat{GCF}$

SOLUTION:

Here, $\widehat{GCF}$ is the longest arc connecting the points G and F on $\odot B.$ Therefore, it is a major arc.
Arc GCF shares the same endpoints as minor arc GF.

$$\begin{aligned} m(\text{arc}\,GCF) &= 360 - m(\text{arc}\,GF) && \text{Measure of Major Arc Rule} \\ &= 360 - 35 && m(\text{arc}\,GF) = m\angle GBF \\ &= 325 && \text{Simplify.} \end{aligned}$$

Therefore, the measure of arc GCF is 325.

21. $m\widehat{ACD}$

SOLUTION:

Here, $\overline{AD}$ is a diameter. Therefore, $\widehat{ACD}$ is a semicircle.
The measure of a semicircle is 180, so m(arc ACD) = 180.

22. $m\widehat{AG}$

SOLUTION:

Here, $\widehat{AG}$ is the shortest arc connecting the points A and G on $\odot B.$ Therefore, it is a minor arc.

The measure of a minor arc is equal to the measure of its related central angle.

$$\begin{aligned} m\angle ABG &= m\angle CBD && \text{Vertical angles are congruent.} \\ m\angle ABG &= 55 && \text{Substitution} \\ m(\text{arc}\,AG) &= m\angle ABG && \text{Measure of Minor Arc Rule} \\ m(\text{arc}\,AG) &= 55 && \text{Substitution} \end{aligned}$$

Therefore, the measure of arc AG is 55.

23. $m\widehat{ACF}$

SOLUTION:

Here, $\widehat{ACF}$ is the longest arc connecting the points A and F on $\odot B$. Therefore, it is a major arc.
Major arc ACF shares the same endpoints as minor arc AF, so $m(\text{arc } ACF) = 360 - m(\text{arc } AF)$.
Since $\angle ABG$ and $\angle CBD$ are vertical angles, $m\angle ABG = m\angle CBD$ or 55.

$$\begin{aligned} m(\text{arc } AF) &= m(\text{arc } AG) + m(\text{arc } GF) && \text{Arc Addition Postulate} \\ &= 55 + 35 && m(\text{arc } AG) = m\angle ABG,\ m(\text{arc } GF) = m\angle GBF \\ &= 90 && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} m(\text{arc } ACF) &= 360 - m(\text{arc } AF) && \text{Measure of Major Arc Rule} \\ &= 360 - 90 && \text{Substitution} \\ &= 270 && \text{Simplify.} \end{aligned}$$

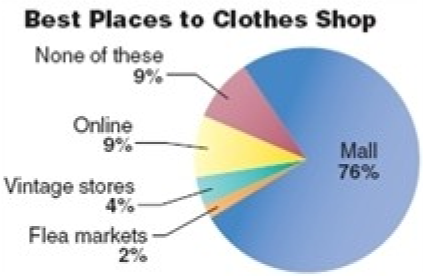
Therefore, the measure of arc ACF is 270.

24. **SHOPPING** The graph shows the results of a survey in which teens were asked where the best place was to shop for clothes.

a. What would be the arc measures associated with the mall and vintage stores categories?

b. Describe the kinds of arcs associated with the category "Mall"and category "None of these".

c. Are there any congruent arcs in this graph? Explain.



SOLUTION:

a. The measure of the arc is equal to the measure of the central angle. The mall contributes 76% and the vintage stores contribute 4% of the total shopping. Find the 76% of 360 to find the central angle of the arc associated with the malls.

$$\frac{76}{100} \cdot 360 = 273.6$$

Find the 4% of 360 to find the central angle of the arc associated with the vintage stores.

$$\frac{4}{100} \cdot 360 = 14.4$$

b. The arc associated with the mall has a measure of 273.6. So, it is a major arc. The arc associated with none of these has a measure of 9% of 360 or 32.4. So, it is a minor arc.

c. Yes; the arcs associated with the online and none of these categories have the same arc measure since each category accounts for the same percentage of the circle, 9%.

25. **FOOD** The table shows the results of a survey in which Americans were asked how long food could be on the floor and still be safe to eat.

a. If you were to construct a circle graph of this information, what would be the arc measures associated with the first two categories?

b. Describe the kind of arcs associated with the first category and the last category.

c. Are there any congruent arcs in this graph? Explain.

Dropped Food	
Do you eat food dropped on the floor?	
Not safe to eat	78%
Three-second rule*	10%
Five-second rule*	8%
Ten-second rule*	4%

Source: American Diabetic Association

* The length of time the food is on the floor.

SOLUTION:

a. The measure of the arc is equal to the measure of the central angle. The "not so safe" category contributes 78% and the "three-second rule" contributes 10% in the supporters in the survey. Find the 78% of 360 to find the central angle of the arc associated with the not so safe category.

$0.78 \cdot 360 = 280.8$

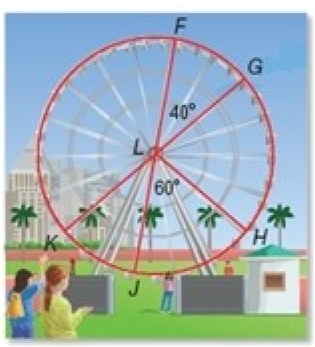
Find the 10% of 360 to find the central angle of the arc associated with the three-second rule category.

$0.10 \cdot 360 = 36$

b. The arc corresponding to not safe to eat category measures 280.8, so it is a major arc. Similarly, the arc corresponding to the ten-second rule measures 4% of 360 or 14.4, so it is a minor arc.

c. No; no categories share the same percentage of the circle.

ENTERTAINMENT Use the Ferris wheel shown to find each measure.



26. $m\widehat{FG}$

SOLUTION:

The measure of the arc is equal to the measure of the central angle. We have, $m\angle FLG = 40.$

Therefore, $m\widehat{FG} = 40.$

27. $m\widehat{JH}$

SOLUTION:

The measure of the arc is equal to the measure of the central angle. We have, $m\angle JLH = 60.$

Therefore, $m\widehat{JH} = 60.$

28. $m\widehat{JKF}$

SOLUTION:

Here, $\overline{JF}$ is a diameter. Therefore, $\widehat{JKF}$ is a semicircle and $m\widehat{JKF} = 180.$

29. $m\widehat{JFH}$

SOLUTION:

Arc *JFH* is a major arc.

$$\begin{aligned} m(\text{arc}\,JFH) &= 360 - m(\text{arc}\,JH) && \text{Measure of Major Arc Rule} \\ &= 360 - 60 && m(\text{arc}\,JH) = m\angle JLH \\ &= 300 && \text{Simplify.} \end{aligned}$$

Therefore, the measure of arc *JFH* is 300.

30. $m\widehat{GHF}$

SOLUTION:

Arc *GHF* is a major arc.

$$\begin{aligned} m(\text{arc}\,GHF) &= 360 - m(\text{arc}\,GF) && \text{Measure of Major Arc Rule} \\ &= 360 - 40 && m(\text{arc}\,GF) = m\angle GLF \\ &= 320 && \text{Simplify.} \end{aligned}$$

Therefore, the measure of arc *GHF* is 320.

31. $m\widehat{GHK}$

SOLUTION:

Here, $\overline{GK}$ is a diameter. Therefore, $\widehat{GHK}$ is a semicircle and $m\widehat{GHK} = 180.$

32. $m\widehat{HK}$

SOLUTION:

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. First find $m\angle JLK$ and then use the Arc Addition Postulate.

$$\begin{aligned} m\angle JLK &= m\angle FLG && \text{Vertical angles are congruent.} \\ m\angle JLK &= 40 && \text{Substitutition} \end{aligned}$$

$$\begin{aligned} m(\text{arc}\,HK) &= m(\text{arc}\,HJ) + m(\text{arc}\,JK) && \text{Arc Addition Postulate} \\ &= 60 + 40 && m(\text{arc}\,HJ) = m\angle HLJ,\ m(arc\,JK) = m\angle JLK \\ &= 100 && \text{Simplify.} \end{aligned}$$

Therefore, the measure of arc *HK* is 100.

33. $m\widehat{JKG}$

SOLUTION:

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. $\overline{JF}$ is a diameter, so arc *JF* is a semicircle and has a measure of 180.

$$\begin{aligned} m(\text{arc } JKG) &= m(\text{arc } JF) + m(\text{arc } FG) && \text{Arc Addition Postulate} \\ &= 180 + 40 && m(\text{arc } FG) = m\angle FLG \\ &= 220 && \text{Simplify.} \end{aligned}$$

Therefore, the measure of arc *JKG* is 220.

34. $m\widehat{KFH}$

SOLUTION:

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. $\overline{KG}$ and $\overline{FJ}$ are diameters, so arc *KG* and arc *FJ* are semicircles with measures of 180. Find $m\angle GLH$ and then use the Arc Addition Postulate.

$$\begin{aligned} m\angle FLG + m\angle GLH + m\angle HLJ &= m\angle FLJ && \text{Angle Addition Postulate} \\ 40 + m\angle GLH + 60 &= 180 && \text{Substitution} \\ m\angle GLH + 100 &= 180 && \text{Simplify.} \\ m\angle GLH &= 80 && \text{Subtract 100 from each side.} \end{aligned}$$

$$\begin{aligned} m(\text{arc } KFH) &= m(\text{arc } KG) + m(\text{arc } GH) && \text{Arc Addition Postulate} \\ &= 180 + 80 && m(\text{arc } GH) = m\angle GLH \\ &= 260 && \text{Simplify.} \end{aligned}$$

Therefore, the measure of arc *KFH* is 260.

35. $m\widehat{HGF}$

SOLUTION:

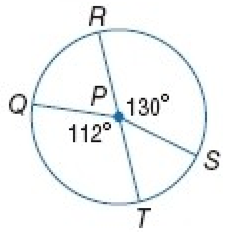
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. Use semicircle *FJ* to find $m\angle HLG$ and then use the Arc Addition Postulate.

$$\begin{aligned} m\angle FLG + m\angle HLG + m\angle HLJ &= m\angle FLJ && \text{Angle Addition Postulate} \\ 40 + m\angle HLG + 60 &= 180 && \text{Substitution} \\ m\angle HLG + 100 &= 180 && \text{Simplify.} \\ m\angle HLG &= 80 && \text{Subtract 100 from each side.} \end{aligned}$$

$$\begin{aligned} m(\text{arc } HGF) &= m(\text{arc } HG) + m(\text{arc } GK) && \text{Arc Addition Postulate} \\ &= 80 + 40 && m(\text{arc } HG) = m\angle HLG, m(arc\ GK) = m\angle GLK \\ &= 120 && \text{Simplify.} \end{aligned}$$

Therefore, the measure of arc *HGF* is 120.

Use $\odot P$ to find the length of each arc. Round to the nearest hundredth.



36. $\overset{\frown}{RS}$, if the radius is 2 inches

SOLUTION:

Use the arc length equation with r = 2 inches and m(arc RS) = $m\angle RPS$ or 130.

$$\begin{aligned} \ell &= \frac{x}{360} \cdot 2\pi r && \text{Arc Length Equation} \\ &= \frac{130}{360} \cdot 2\pi(2) && \text{Substitution} \\ &\approx 4.54 && \text{Use a calculator.} \end{aligned}$$

Therefore, the length of arc RS is about 4.54 inches.

37. $\overset{\frown}{QT}$, if the diameter is 9 centimeters

SOLUTION:

Use the arc length equation with $r = \frac{1}{2}(9)$ or 4.5 centimeters and m(arc QT) = $m\angle QPT$ or 112.

$$\begin{aligned} \ell &= \frac{x}{360} \cdot 2\pi r && \text{Arc Length Equation} \\ &= \frac{112}{360} \cdot 2\pi(4.5) && \text{Substitution} \\ &\approx 8.80 && \text{Use a calculator.} \end{aligned}$$

Therefore, the length of arc QT is about 8.80 centimeters.

38. $\overset{\frown}{QR}$, if PS = 4 millimeters

SOLUTION:

$\overline{PS}$ is a radius of $\odot P$ and $\overline{RT}$ is a diameter. Use the Arc Addition Postulate to find m(arc QR).

$$\begin{aligned} m(\text{arc}\,RT) &= m(\text{arc}\,QR) + m(\text{arc}\,QT) && \text{Arc Addition Postulate} \\ 180 &= m(\text{arc}\,QR) + 112 && \text{arc}\,RT \text{ is a semicircle, } m(\text{arc}\,QT) = m\angle QPT \\ 68 &= m(\text{arc}\,QR) && \text{Simplify.} \end{aligned}$$

Use the arc length equation with r = 4 millimeters and x = m(arc RQ) or 68.

$$\begin{aligned} \ell &= \frac{x}{360} \cdot 2\pi r && \text{Arc Length Equation} \\ &= \frac{68}{360} \cdot 2\pi(4) && \text{Substitution} \\ &\approx 4.75 && \text{Use a calculator.} \end{aligned}$$

Therefore, the length of arc QR is about 4.75 millimeters.

39. $\widehat{RS}$, if RT = 15 inches

SOLUTION:

$\overline{RT}$ is a diameter, so $r = \frac{1}{2}(15)$ or 7.5 inches. Use the arc length equation with $x = m(\text{arc } RS)$ or 130.

$$\begin{aligned} \ell &= \frac{x}{360} \cdot 2\pi r && \text{Arc Length Equation} \\ &= \frac{130}{360} \cdot 2\pi(7.5) && \text{Substitution} \\ &\approx 17.02 && \text{Use a calculator.} \end{aligned}$$

Therefore, the length of arc *RS* is about 17.02 inches.

40. $\widehat{QRS}$, if RT = 11 feet

SOLUTION:

Use the Arc Addition Postulate to find the $m(\text{arc } QR)$.

$$\begin{aligned} m(\text{arc } RT) &= m(\text{arc } QR) + m(\text{arc } QT) && \text{Arc Addition Postulate} \\ 180 &= m(\text{arc } QR) + 112 && \text{arc } RT \text{ is a semicircle, } m(\text{arc } QT) = m\angle QPT \\ 68 &= m(\text{arc } QR) && \text{Simplify.} \end{aligned}$$

Use the Arc Addition Postulate to find $m(\text{arc } QRS)$.

$$\begin{aligned} m(\text{arc } QRS) &= m(\text{arc } QR) + m(\text{arc } RS) && \text{Arc Addition Postulate} \\ &= 68 + 130 && m(\text{arc } RS) = m\angle RPS \\ &= 198 && \text{Simplify.} \end{aligned}$$

Use the arc length equation with $r = \frac{1}{2}(RT)$ or 5.5 feet and $x = m(\text{arc } QRS)$ or 198.

$$\begin{aligned} \ell &= \frac{x}{360} \cdot 2\pi r && \text{Arc Length Equation} \\ &= \frac{198}{360} \cdot 2\pi(5.5) && \text{Substitution} \\ &\approx 19.01 && \text{Use a calculator.} \end{aligned}$$

Therefore, the length of arc *QRS* is about 19.01 feet.

41. $\widehat{RTS}$, if PQ = 3 meters

SOLUTION:

Arc *RTS* is a major arc that shares the same endpoints as minor arc *RS*.

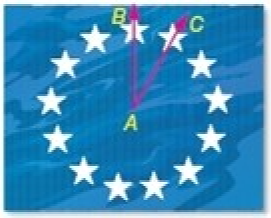
$$\begin{aligned} m(\text{arc } RTS) &= 360 - m(\text{arc } RS) && \text{Measure of Major Arc Rule} \\ &= 360 - 130 && m(\text{arc } RS) = m\angle RPS \\ &= 230 && \text{Simplify.} \end{aligned}$$

Use the arc length equation with $r = PS$ or 3 meters and $x = m(\text{arc } RTS)$ or 230.

$$\begin{aligned} \ell &= \frac{x}{360} \cdot 2\pi r && \text{Arc Length Equation} \\ &= \frac{230}{360} \cdot 2\pi(3) && \text{Substitution} \\ &\approx 12.04 && \text{Use a calculator.} \end{aligned}$$

Therefore, the length of arc *RTS* is about 12.04 meters.

HISTORY The figure shows the stars in the Betsy Ross flag referenced at the beginning of the lesson.



42. What is the measure of central angle *A*? Explain how you determined your answer.

SOLUTION:

There are 13 stars arranged in a circular way equidistant from each other. So, the measure of the central angle of the arc joining any two consecutive stars will be equal to $\frac{360}{13} \approx 27.7.$

43. If the diameter of the circle were doubled, what would be the effect on the arc length from the center of one star *B* to the next star *C*?

SOLUTION:

The measure of the arc between any two stars is about 27.7. Let ℓ_1 be the arc length of the original circle and ℓ_2 be the arc length for the circle when the diameter is doubled. Use the arc length equation with a radius of *r* and *x* = 27.7 to find ℓ_1.

$$\begin{aligned} \ell &= \frac{x}{360} \cdot 2\pi r && \text{Arc Length Equation} \\ \ell_1 &= \frac{27.7}{360} \cdot 2\pi(r) && \text{Substitution} \\ \ell_1 &= \frac{27.7}{360} \cdot 2\pi r && \text{Simplify.} \end{aligned}$$

Use the arc length equation with a radius of 2*r* and *x* = 27.7 to find ℓ_2.

$$\begin{aligned} \ell &= \frac{x}{360} \cdot 2\pi r && \text{Arc Length Equation} \\ \ell_2 &= \frac{27.7}{360} \cdot 2\pi(2r) && \text{Substitution} \\ \ell_2 &= 2\left(\frac{27.7}{360} \cdot 2\pi r\right) \text{ or } 2(\ell_1) && \text{Simplify.} \end{aligned}$$

The arc length for the second circle is twice the arc length for the first circle.
Therefore, if the diameter of the circle is doubled, the arc length from the center of star *B* to the center of the next star *C* would double.

44. **FARMS** The *Pizza Farm* in Madera, California, is a circle divided into eight equal slices, as shown at the right. Each "slice" is used for growing or grazing pizza ingredients.
a. What is the total arc measure of the slices containing olives, tomatoes, and peppers?
b. The circle is 125 feet in diameter. What is the arc length of one slice? Round to the nearest hundredth.

SOLUTION:

a. The circle is divided into eight equal slices. So, the measure of the central angle of each slice will be $\frac{360}{8}=45$. Therefore, total arc measure of the slices containing olives, tomatoes, and peppers will be 3(45) = 135.

b. The length of an arc l is given by the formula,

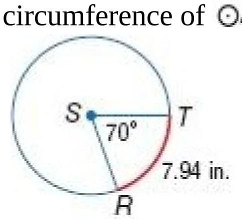
$l=\frac{x}{360}\cdot 2\pi r$ where x is the central angle of the arc l and r is the radius of the circle.

The measure of the central angle of each slice will be $\frac{360}{8}=45$. So, x = 45 and r 62.5 ft. Then,

$$\text{arc length of each slice}=\frac{45}{360}\cdot 2\pi(62.5)\approx 49.09\text{ ft.}$$

CCSS REASONING Find each measure. Round each linear measure to the nearest hundredth and each arc measure to the nearest degree.

45. circumference of $\odot S$

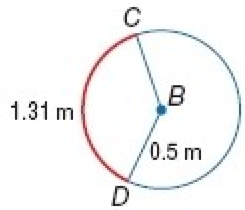


SOLUTION:

The circumference of circle S is given by . Use the arc length equation and solve for the value of .

Therefore, the circumference of circle S is about 40.83 inches.

46. $m\widehat{CD}$



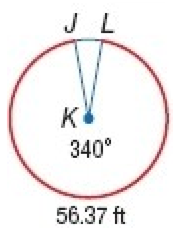
SOLUTION:

The radius of circle *B* is 0.5 meters and the length of arc *CD* is 1.31 meters. Use the arc length equation and solve for *x* to find the measure of arc *CD*.

$$\begin{aligned} \ell &= \frac{x}{360}2\pi r && \text{Arc Length Equation} \\ 1.31 &= \frac{x}{360}2\pi(0.5) && \ell = 1.31, r = 0.5 \\ 1.31 &= \frac{x\pi}{360} && \text{Simplify.} \\ 150 &\approx x && \text{Multiply each side by } \frac{360}{\pi}. \end{aligned}$$

Therefore, the measure of arc *CD* is about 150°.

47. radius of $\odot K$



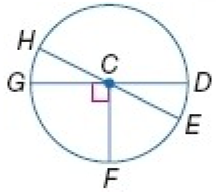
SOLUTION:

The measure of major arc *JL* is 340 and its arc length is 56.37 feet. Use the arc length equation to solve for the radius of circle *K*.

$$\begin{aligned} \ell &= \frac{x}{360}2\pi r && \text{Arc Length Equation} \\ 56.37 &= \frac{340}{360}2\pi r && \ell = 56.37; x = 340 \\ 56.37 &= \frac{680\pi}{360}r && \text{Simplify.} \\ 9.50 &\approx r && \text{Multiply each side by } \frac{360}{680\pi}. \end{aligned}$$

Therefore, the radius of circle *K* is about 9.50 feet.

ALGEBRA In $\odot C$, $m\angle HCG$ = 2*x* and $m\angle HCD$ = 6*x* + 28. Find each measure.



48. $m\widehat{EF}$

SOLUTION:

Here, $\angle HCG$ and $\angle HCD$ form a linear pair. So, the sum of their measures is 180.

$$\begin{aligned} m\angle HCG + m\angle HCD &= 180 && \text{Definition of Linear Pair} \\ (2x) + (6x + 28) &= 180 && \text{Substitution.} \\ 8x &= 152 && \text{Subtract 28 from each side.} \\ x &= 19 && \text{Divide each side by 8.} \end{aligned}$$

So, $m\angle HCG$ = 2(19) or 38 and $m\angle HCD$ = 6(19) + 28 or 142.

$\overline{HE}$ is a diameter of circle *C*, so arc *HFE* is a semicircle and has a measure of 180.

$$\begin{aligned} m(\text{arc}\,HG) + m(\text{arc}\,GF) + m(\text{arc}\,EF) &= m(\text{arc}\,HFE) && \text{Arc Addition Postulate} \\ 38 + 90 + m(\text{arc}\,EF) &= 180 && m(\text{arc}\,HG) = m\angle HCG,\ m(arc\,GF) = m\angle GCF \\ m(\text{arc}\,EF) &= 52 && \text{Subtract 128 from each side.} \end{aligned}$$

Therefore, the measure of arc *EF* is 52.

49. $m\widehat{HD}$

SOLUTION:

Here, $\angle HCG$ and $\angle HCD$ form a linear pair. So, the sum of their measures is 180.

$$\begin{aligned} m\angle HCG + m\angle HCD &= 180 && \text{Definition of Linear Pair} \\ (2x) + (6x + 28) &= 180 && \text{Substitution.} \\ 8x &= 152 && \text{Subtract 28 from each side.} \\ x &= 19 && \text{Divide each side by 8.} \end{aligned}$$

So, $m\angle HCG$ = 2(19) or 38 and $m\angle HCD$ = 6(19) + 28 or 142.

The measure of an arc is equal to the measure of its related central angle.

Therefore, *m*(arc *HD*) = $m\angle HCD$ or 142.

50. $m\widehat{HGF}$

SOLUTION:

Here, $\angle HCG$ and $\angle HCD$ form a linear pair. So, the sum of their measures is 180.

$$\begin{aligned} m\angle HCG + m\angle HCD &= 180 && \text{Definition of Linear Pair} \\ (2x) + (6x+28) &= 180 && \text{Substitution.} \\ 8x &= 152 && \text{Subtract 28 from each side.} \\ x &= 19 && \text{Divide each side by 8.} \end{aligned}$$

So, $m\angle HCG$ = 2(19) or 38 and $m\angle HCD$ = 6(19) + 28 or 142.
Use the Arc Addition Postulate to find the measure of arc *HGF*.

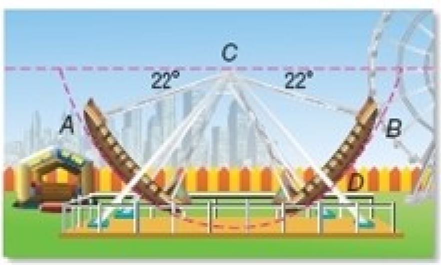
$$\begin{aligned} m(\text{arc}\,HGF) &= m(\text{arc}\,HG) + m(\text{arc}\,GF) && \text{Arc Addition Postulate} \\ &= 38 + 90 && m(\text{arc}\,HG) = m\angle HCG,\ m(arc\,GF) = m\angle GCF \\ &= 128 && \text{Simplify.} \end{aligned}$$

Therefore, the measure of arc *HGF* is 128.

51. **RIDES** A pirate ship ride follows a semi-circular path, as shown in the diagram.

a. What is $m\widehat{AB}$?

b. If *CD* = 62 feet, what is the length of $\widehat{AB}$? Round to the nearest hundredth.



SOLUTION:

a. From the figure, $\widehat{AB}$ is 22 + 22 = 44° less than the semi circle centered at *C*. Therefore, $m\widehat{AB} = 180 - 44 = 136.$

b. The length of an arc *l* is given by the formula,

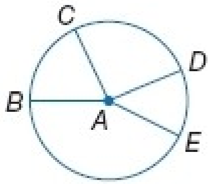
$l = \frac{x}{360} \cdot 2\pi r$ where *x* is the central angle of the arc *l* and *r* is the radius of the circle.

Here, *x* = 136 and *r* = 62. Use the formula.

$$\begin{aligned} \text{length of } \widehat{AB} &= \frac{136}{360}(2\pi \cdot 62) \\ &\approx 147.17 \text{ ft} \end{aligned}$$

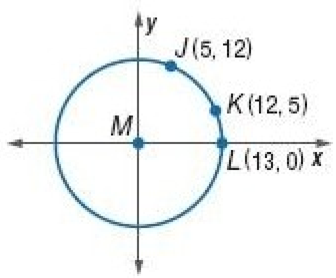
10-2 Measuring Angles and Arcs

52. **PROOF** Write a two-column proof of Theorem 10.1.
Given: $\angle BAC \cong \angle DAE$
Prove: $\widehat{BC} \cong \widehat{DE}$



SOLUTION:
Proof:
Statements (Reasons)
1. $\angle BAC \cong \angle DAE$ (Given)
2. $m\angle BAC = m\angle DAE$ (Definition of $\cong \angle$s)
3. $m\angle BAC = m\widehat{BC}$, $m\angle DAE = m\widehat{DE}$ (Definition of arc measure)
4. $m\widehat{BC} = m\widehat{DE}$ (Substitution)
5. $\widehat{BC} \cong \widehat{DE}$ (Definition of $\cong$ arcs)

53. **COORDINATE GEOMETRY** In the graph, point M is located at the origin. Find each measure in $\odot M$. Round each linear measure to the nearest hundredth and each arc measure to the nearest tenth degree.



a. $m\widehat{JL}$
b. $m\widehat{KL}$
c. $m\widehat{JK}$
d. length of $\widehat{JL}$
e. length of $\widehat{JK}$

SOLUTION:
a. The measure of arc *JL* equals the measure of the related central angle *JML*. Construct a right triangle by drawing $\overline{MJ}$ and a perpendicular segment from *J* to the *x*-axis. The legs of the right triangle will have lengths of 5 and 12.
Use a trigonometric ratio to find the angle of the triangle at M which equals the measure of central angle *JML*.

$$\begin{aligned} \tan\theta &= \frac{opp}{adj} && \text{Definition of Tangent} \\ \tan(\angle JML) &= \frac{12}{5} && \text{Substitution} \\ m\angle JML &= \tan^{-1}\left(\frac{12}{5}\right) && \text{Use the inverse tangent ratio.} \\ m\angle JML &\approx 67.4 && \text{Use a calculator.} \end{aligned}$$

Therefore, $m\widehat{JL} = 67.4$.

b. The measure of arc *KL* equals the measure of the related central angle *KML*. Construct a right triangle by drawing $\overline{MK}$ and a perpendicular segment from *K* to the *x*-axis. The legs of the right triangle will have lengths of 12 and 5.
Use a trigonometric ratio to find the angle of the triangle at *M* which equals the measure of central angle *KML*.

$$\begin{aligned} \tan\theta &= \frac{opp}{adj} && \text{Definition of Tangent} \\ \tan(\angle KML) &= \frac{5}{12} && \text{Substitution} \\ m\angle KML &= \tan^{-1}\left(\frac{5}{12}\right) && \text{Use the inverse tangent ratio.} \\ m\angle KML &\approx 22.6 && \text{Use a calculator.} \end{aligned}$$

Therefore, $m\widehat{KL} = 22.6$.
c. Use the Arc Addition Postulate to find *m*(arc *JK*).

$$\begin{aligned} m(\text{arc } JL) &= m(\text{arc } JK) + m(\text{arc } KL) && \text{Arc Addition Postulate} \\ 67.4 &= m(\text{arc } JK) + 22.6 && \text{Substitute} \\ 44.8 &= m(\text{arc } JK) && \text{Subtract 22.6 from each side.} \end{aligned}$$

So, the measure of arc *JK* is 44.8.
d. Use the right triangle from part **a** and find the length of $\overline{MJ}$ which is a radius of circle *M*.

$$\begin{aligned} c^2 &= a^2 + b^2 && \text{Pythagorean Theorem} \\ MJ^2 &= 5^2 + 12^2 && \text{Substitution} \\ MJ^2 &= 169 && \text{Simplify.} \\ MJ &= 13 && \text{Take the positive square root of each side.} \end{aligned}$$

Use the arc length equation with $r = 13$ and $x = m(\text{arc } JL)$ or 67.4.

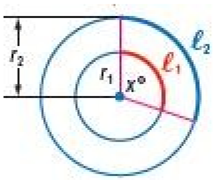
$$\begin{aligned} \ell &= \frac{x}{360}\cdot 2\pi r && \text{Arc Length Equation} \\ &= \frac{67.4}{360}\cdot 2\pi(13) && \text{Substitution} \\ &\approx 15.29 && \text{Use a calculator.} \end{aligned}$$

Therefore, the length of arc *JL* is about 15.29 units.

e. Use the arc length equation with $r = 13$ and $x = m(\text{arc } JK)$ or 44.8.

$$\begin{aligned} \ell &= \frac{x}{360}\cdot 2\pi r && \text{Arc Length Equation} \\ &= \frac{44.8}{360}\cdot 2\pi(13) && \text{Substitution} \\ &\approx 10.16 && \text{Use a calculator.} \end{aligned}$$

Therefore, the length of arc *JK* is about 10.16 units.

54. 

ARC LENGTH AND RADIAN MEASURE In this problem, you will use concentric circles to show that the length of the arc intercepted by a central angle of a circle is dependent on the circle's radius.

a. Compare the measures of arc ℓ_1 and arc ℓ_2. Then compare the lengths of arc ℓ_1 and arc ℓ_2. What do these two comparisons suggest?

b. Use similarity transformations (dilations) to explain why the length of an arc ℓ intercepted by a central angle of a circle is proportional to the circle's radius r. That is, explain why we can say that for this diagram, $\frac{\ell_1}{r_1} = \frac{\ell_2}{r_2}$.

c. Write expressions for the lengths of arcs ℓ_1 and ℓ_2. Use these expressions to identify the constant of proportionality k in $\ell = kr$.

d. The expression that you wrote for k in part **c** gives the radian measure of an angle. Use it to find the radian measure of an angle measuring 90°.

SOLUTION:

a. By definition, the measure of an arc is equal to the measure of its related central angle. So, m(arc ℓ_1) = x and m (arc ℓ_2) = x. By the transitive property, m(arc ℓ_1) = m(arc ℓ_2). Using the arc length equation, $\ell_1 = \frac{x}{360}(2\pi r_1)$ and $\ell_2 = \frac{x}{360}(2\pi r_2)$. Since $r_1 < r_2$, then $\ell_1 < \ell_2$.
These comparisons suggest that arc measure is not affected by the size of the circle, but arc length is affected.
b. Since all circles are similar, the larger circle is a dilation of the smaller by some factor k, so $r_2 = kr_1$ or $k = \frac{r_2}{r_1}$. Likewise, the arc intercepted on the larger circle is a dilation of the arc intercepted on the smaller circle, so $\ell_2 = k\ell_1$ or $k = \frac{\ell_2}{\ell_1}$. Then substitute for k.

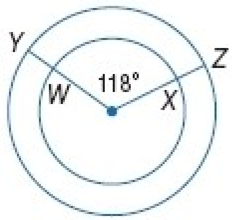
$$\frac{r_2}{r_1} = \frac{\ell_2}{\ell_1} \quad \text{Substitute}$$

$$r_2\ell_1 = r_1\ell_2 \quad \text{Cross products are equal.}$$

$$\frac{\ell_1}{r_1} = \frac{\ell_2}{r_2} \quad \text{Divide each side by } r_1 r_2.$$

c. $\ell_1 = \frac{x}{360}(2\pi r_1)$ or $\frac{\pi x}{180}(r_1)$ and $\ell_2 = \frac{x}{360}(2\pi r_2)$ or $\frac{\pi x}{180}(r_2)$; Therefore, for $\ell = kr$, $k = \frac{\pi x}{180}$.
d. For an angle measuring 90, x = 90. Then, $k = \frac{90\pi}{180}$ or $\frac{\pi}{2}$.

55. **ERROR ANALYSIS** Brody says that $\widehat{WX}$ and $\widehat{YZ}$ are congruent since their central angles have the same measure. Selena says they are not congruent. Is either of them correct? Explain your reasoning.



SOLUTION:

Brody has incorrectly applied Theorem 10.1. The arcs are congruent if and only if their central angles are congruent and the arcs and angles are in the same circle or congruent circles. The circles containing arc *WX* and arc *YZ* are not congruent because they do not have congruent radii. The arcs will have the same degree measure but will have different arc lengths. So, the arcs are not congruent. Therefore, Selena is correct.

REASONING Determine whether each statement is *sometimes, always,* or *never* true. Explain your reasoning.

56. The measure of a minor arc is less than 180.

SOLUTION:

By definition, an arc that measures less than 180 is a minor arc. Therefore, the statement is *always* true.

57. If a central angle is obtuse, its corresponding arc is a major arc.

SOLUTION:

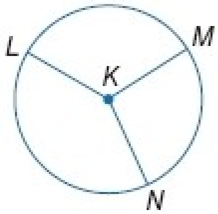
Obtuse angles intersect arcs between $90°$ and $180°$. So, the corresponding arc will measure less than $180°$. Therefore, the statement is *never* true.

58. The sum of the measures of adjacent arcs of a circle depends on the measure of the radius.

SOLUTION:

Postulate 10.1 says that the measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. The measure of each arc would equal the measure of its related central angle. The radius of the circle does not depend on the radius of the circle. Therefore, the statement is *never* true.

59. **CHALLENGE** The measures of $\widehat{LM}$, $\widehat{MN}$, and $\widehat{NL}$ are in the ratio 5:3:4. Find the measure of each arc.



SOLUTION:

If the measures of arc *LM*, arc *MN*, and arc *NL* are in the ratio 5:3:4, then their measures are a multiple, *x*, of these numbers. So, *m*(arc *LM*) = 5*x*, *m*(arc *MN*) = 3*x*, and *m*(arc *MN*) = 4*x*. The arcs are adjacent and form the entire circle, so their sum is 360.

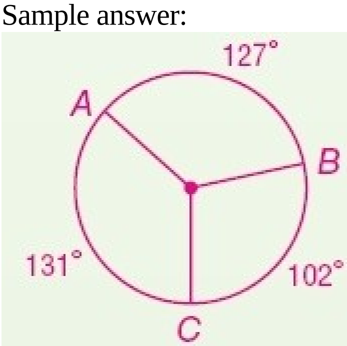
$$
\begin{aligned}
5x + 3x + 4x &= 360 && \text{Sum is 360.}\\
12x &= 360 && \text{Simplify.}\\
x &= 30 && \text{Divide each side by 12.}
\end{aligned}
$$

Therefore, *m*(arc *LM*) = 5(30) or 150, *m*(arc *MN*) = 3(30) or 90, and *m*(arc *NL*) = 4(30) or 120.

60. **OPEN ENDED** Draw a circle and locate three points on the circle. Estimate the measures of the three nonoverlapping arcs that are formed. Then use a protractor to find the measure of each arc. Label your circle with the arc measures.

SOLUTION:

Sample answer:



61. **CHALLENGE** The time shown on an analog clock is 8:10. What is the measure of the angle formed by the hands of the clock?

SOLUTION:

At 8:00, the minute hand of the clock will point at 12 and the hour hand at 8. At 8:10, the minute hand will point at 2 and the hour hand will have moved $\frac{10}{60}$ or $\frac{1}{6}$ of the way between 8 and 9.

The sum of the measures of the central angles of a circle with no interior points in common is 360. The numbers on an analog clock divide it into 12 equal arcs and the central angle related to each arc between consecutive numbers has a measure of $\frac{360}{12}$ or 30.

To find the measure of the angle formed by the hands, find the sum of the angles each hand makes with 12.

At 8:10, the minute hand is at 2 which is two arcs of 30 from 12 or a measure of 60.

When the hour hand is on 8, the angle between the hour hand and 12 is equal to four arcs of 30 or 120. At 8:10, the hour hand has moved $\frac{1}{6}$ of the way from 8 to 9. Since the arc between 8 and 9 measures 30, the angle between the hand and 8 is $\frac{1}{6}(30)$ or 5. The angle between the hour hand and 12 is 120 - 5 or 115.

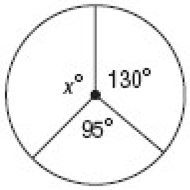
The sum of the measures of the angles between the hands and 12 is 60 + 115 or 175.
Therefore, the measure of the angle formed by the hands of the clock at 8:10 is 175.

62. **WRITING IN MATH** Describe the three different types of arcs in a circle and the method for finding the measure of each one.

SOLUTION:

Sample answer: Minor arc, major arc, semicircle; the measure of a minor arc equals the measure of the corresponding central angle. The measure of a major arc equals 360 minus the measure of the minor arc with the same endpoints. The measure of a semicircle is 180 since it is an arc with endpoints that lied on a diameter.

63. What is the value of x?



A 120
B 135
C 145
D 160

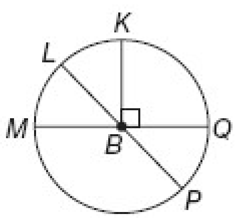
SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360.

$$x + 95 + 130 = 360 \quad \text{Sum of Central Angles}$$
$$x + 225 = 360 \quad \text{Simplify.}$$
$$x = 135 \quad \text{Subtract 225 from each side.}$$

Therefore, the correct choice is B.

64. **SHORT RESPONSE** In $\odot B$, $m\angle LBM = 3x$ and $m\angle LBQ = 4x + 61$. What is the measure of $\angle PBQ$?



SOLUTION:

Here, $\angle LBM$ and $\angle LBQ$ form a linear pair. So, the sum of their measures is 180.
$3x + 4x + 61 = 180$
$7x = 119$
$x = 17$
$m\angle PBQ = 3(17)$ or 51
Since $\angle LBM$ and $\angle PBQ$ are vertical angles, $m\angle LBM = m\angle PBQ$.
Therefore, the measure of $\angle LBM$ is 51.

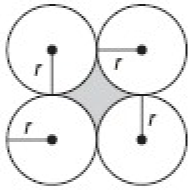
65. **ALGEBRA** A rectangle's width is represented by x and its length by y. Which expression best represents the area of the rectangle if the length and width are tripled?

F $3xy$
G $3(xy)^2$
H $9xy$
J $(xy)^3$

SOLUTION:

The formula for the area of a rectangle is $A = \ell w$. The area of a rectangle with a length of y and a width of x is $A = xy$. When the dimensions are tripled, the length becomes $3y$ and width becomes $3x$. The area of the new rectangle is given by $A = (3x)(3y)$ or $9xy$.
Therefore, the correct choice is H.

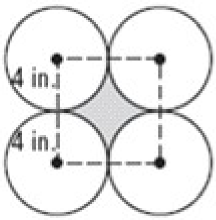
66. **SAT/ACT** What is the area of the shaded region if *r* = 4?



A $64 - 16\pi$
B $16 - 16\pi$
C $16 - 8\pi$
D $64 - 8\pi$
E $64\pi - 16$

SOLUTION:

A square with sides of 8 units can be drawn using the centers of the four circles as vertices. The shaded area is the difference between the areas of the square and the total of the areas of the four quarter sections of the circles formed by the square.



Find the area of the square and the quarter section of the circles.

$A_{\text{square}} = s^2$ $\qquad$ $A_{\text{quarter circle}} = \frac{\pi r^2}{4}$

$A_{\text{square}} = 8^2 \text{ or } 64$ $\qquad$ $A_{\text{quarter circle}} = \frac{\pi(4)^2}{4} \text{ or } 4\pi$

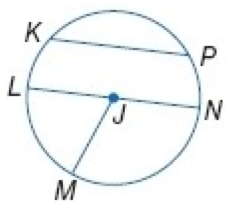
Subtract the area of the four quarter circle sections from the square to find the area of the shaded region.

$A_{\text{shaded}} = 64 - 4(4\pi)$

$A_{\text{shaded}} = 64 - 16\pi$

Therefore, the correct choice is A.

Refer to $\odot J$.



67. Name the center of the circle.

SOLUTION:

Since the circle is named circle *J*, it has a center at *J*.

68. Identify a chord that is also a diameter.

SOLUTION:

A diameter of a circle is a chord that passes through the center and is made up of collinear radii. $\overline{LN}$ passes through the center, so it is a diameter.

69. If $LN = 12.4$, what is JM?

SOLUTION:

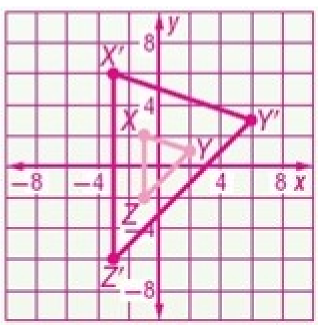
Here, JM is a radius and LN is a diameter. The radius is half the diameter. Therefore, $JM = \frac{LN}{2} = 6.2$ units.

Graph the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor.

70. $X(-1, 2)$, $Y(2, 1)$, $Z(-1, -2)$; $r = 3$

SOLUTION:

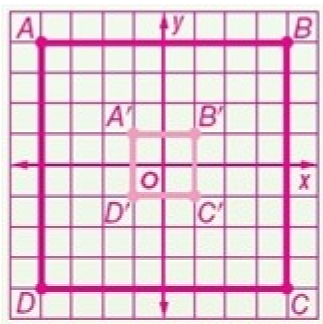
Multiply the x- and y-coordinates of each vertex by the scale factor k. That is, $(x, y) \rightarrow (kx, ky)$.
Here multiply the x- and y-coordinates by the scale factor 3.
$X(-1, 2) \rightarrow X'(-3, 6)$
$Y(2, 1) \rightarrow Y'(6, 3)$
$Z(-1, -2) \rightarrow Z'(-3, -6)$



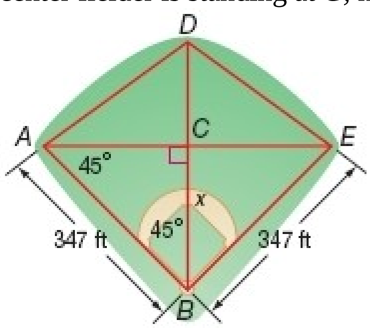
71. $A(-4, 4)$, $B(4, 4)$, $C(4, -4)$, $D(-4, -4)$; $r = 0.25$

SOLUTION:

Multiply the x- and y-coordinates of each vertex by the scale factor k. That is, $(x, y) \rightarrow (kx, ky)$.
Here multiply the x- and y- coordinates by the scale factor 0.25.
$A(-4, 4) \rightarrow A'(-1, 1)$
$B(4, 4) \rightarrow B'(1, 1)$
$C(4, -4) \rightarrow C'(1, -1)$
$D(-4, -4) \rightarrow D'(-1, -1)$



72. **BASEBALL** The diagram shows some dimensions of Comiskey Park in Chicago, Illinois. $\overline{BD}$ is a segment from home plate to dead center field, and $\overline{AE}$ is a segment from the left field foul pole to the right field foul pole. If the center fielder is standing at *C*, how far is he from home plate?



SOLUTION:

Here, ΔABC is a 45°- 45°- 90° triangle and the length of leg $\overline{BC}$ is the distance from the center fielder to home plate. Use Theorem 8.8 to find *x*.

$$h = \ell\sqrt{2} \quad \text{Theorem 8.8}$$

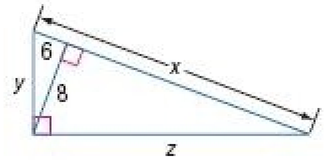
$$347 = x\sqrt{2} \quad \text{Substitution}$$

$$\frac{347}{\sqrt{2}} = x \quad \text{Divide each side by } \sqrt{2}.$$

$$x = \frac{347\sqrt{2}}{2} \quad \text{Rationalize the denominator.}$$

Therefore, the center fielder is standing $\frac{347\sqrt{2}}{2}$ or about 245.4 feet away from home plate.

Find *x*, *y*, and *z*.

73. 

SOLUTION:

Since 8 is the measure of the altitude drawn to the hypotenuse of the large right triangle, 8 is the geometric mean of the lengths of the two segments that make up the hypotenuse, 6 and *x* - 6.

$$\frac{6}{8} = \frac{8}{x-6} \quad \text{Geometric Mean (Altitude) Theorem}$$
$$6x - 36 = 64 \quad \text{Cross Products are equal.}$$
$$6x = 100 \quad \text{Add 36 to each side.}$$
$$x = \frac{50}{3} \quad \text{Divide each side by 6.}$$

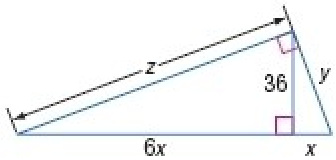
Since y is the measure of a leg of the right triangle, y is the geometric mean of 6 and *x*.

$$\frac{6}{y} = \frac{y}{x} \quad \text{Geometric Mean (Leg) Theorem}$$
$$\frac{6}{y} = \frac{y}{\frac{50}{3}} \quad x = \frac{50}{3}$$
$$y^2 = 100 \quad \text{Cross Products are equal.}$$
$$y = 10 \quad \text{Take the positive square root of each side.}$$

Since z is the measure of the other leg of the right triangle, z is the geometric mean of *x* and *x* - 6.

$$\frac{x-6}{z} = \frac{z}{x} \quad \text{Geometric Mean (Leg) Theorem}$$
$$\frac{\frac{32}{3}}{z} = \frac{z}{\frac{50}{3}} \quad x = \frac{50}{3},\ x - 6 = \frac{50}{3} - 6 \text{ or } \frac{32}{3}$$
$$z^2 = \frac{1600}{9} \quad \text{Cross Products are equal.}$$
$$z = \frac{40}{3} \quad \text{Take the positive square root of each side.}$$

Therefore, $x = \frac{50}{3}$, $y = 10$, and $z = \frac{40}{3}$.

74. 

SOLUTION:

Since 36 is the measure of the altitude drawn to the hypotenuse of the large right triangle, 36 is the geometric mean of the lengths of the two segments that make up the hypotenuse, 6*x* and *x*.

$\frac{6x}{36} = \frac{36}{x}$	Geometric Mean (Altitude) Theorem
$6x^2 = 1296$	Cross Products are equal.
$x^2 = 216$	Divide each side by 6.
$x = 6\sqrt{6}$ or about 14.7	Use a calculator to simplify.

The length of the hypotenuse is *x* + 6*x* or 7*x*. Since *y* is the measure of a leg of the right triangle, *y* is the geometric mean of *x* and 7*x*.

$\frac{x}{y} = \frac{y}{7x}$	Geometric Mean (Leg) Theorem
$\frac{6\sqrt{6}}{y} = \frac{y}{7(6\sqrt{6})}$	$x = 6\sqrt{6}$
$y^2 = 1512$	Cross Products are equal.
$y = 6\sqrt{42}$ or about 38.9	Use a calculator to simplify.

Since z is the measure of the other leg of the right triangle, *z* is the geometric mean of 6*x* and 7*x*.

$\frac{6x}{z} = \frac{z}{7x}$	Geometric Mean (Leg) Theorem
$\frac{6(6\sqrt{6})}{z} = \frac{z}{7(6\sqrt{6})}$	$x = 6\sqrt{6}$
$z^2 = 9072$	Cross Products are equal.
$z = 36\sqrt{7}$ or about 95.2	Use a calculator to simplify.

Therefore, $x = 6\sqrt{6}$ or about 14.7 , $y = 6\sqrt{42}$ or about 38.9, and $z = 36\sqrt{7}$ or about 95.2.

Find *x*.

75. $24^2 + x^2 = 26^2$

SOLUTION:

$$\begin{aligned} 24^2 + x^2 &= 26^2 && \text{Given} \\ 576 + x^2 &= 676 && \text{Simplify.} \\ x^2 &= 100 && \text{Subtract 576 from each side.} \\ x &= 10, -10 && \text{Take the positive and negative square root of each side.} \end{aligned}$$

76. $x^2 + 5^2 = 13^2$

SOLUTION:

$$\begin{aligned} x^2 + 5^2 &= 13^2 && \text{Given} \\ x^2 + 25 &= 169 && \text{Simplify.} \\ x^2 &= 144 && \text{Subtract 25 from each side.} \\ x &= 12, -12 && \text{Take the positive and negative square root of each side.} \end{aligned}$$

77. $30^2 + 35^2 = x^2$

SOLUTION:

$$\begin{aligned} 30^2 + 35^2 &= x^2 && \text{Given} \\ 900 + 1225 &= x^2 && \text{Simplify.} \\ 2125 &= x^2 && \text{Simplify.} \\ 46.1, -46.1 &\approx x && \text{Take the positive and negative square root of each side.} \end{aligned}$$