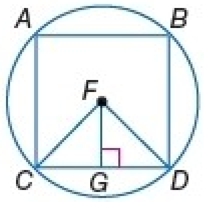
11-4 Areas of Regular Polygons and Composite Figures

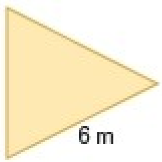
1. In the figure, square *ABDC* is inscribed in $\odot F$. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle.



SOLUTION:

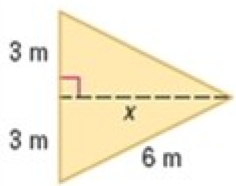
Center: point *F*, radius: $\overline{FD}$, apothem: $\overline{FG}$, central angle: $\overline{CFD}$, A square is a regular polygon with 4 sides. Thus, the measure of each central angle of square *ABCD* is $\frac{360}{4}$ or 90.

Find the area of each regular polygon. Round to the nearest tenth.

2. 

SOLUTION:

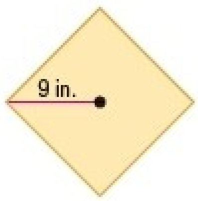
An equilateral triangle has three congruent sides. Draw an altitude and use the Pythagorean Theorem to find the height.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 3^2 + b^2 &= 6^2 \\ b^2 &= 6^2 - 3^2 \\ b^2 &= 36 - 9 \\ b &= \sqrt{27} \\ b &\approx 5.2 \end{aligned}$$

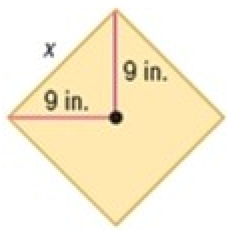
Find the area of the triangle.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &\approx \frac{1}{2}(6)\left(\sqrt{27}\right) \\ &= 15.6 \end{aligned}$$

3. 

SOLUTION:

The polygon is a square. Form a right triangle.



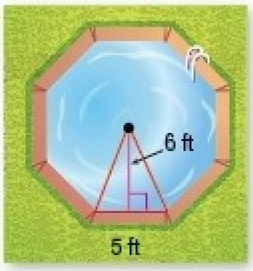
Use the Pythagorean Theorem to find *x*.

$$x^2 = 9^2 + 9^2$$
$$x^2 = 81 + 81$$
$$x^2 = 162$$
$$x = \sqrt{162}$$

Find the area of the square.

$$\begin{aligned} A &= x^2 \\ &= \left(\sqrt{162}\right)^2 \\ &= 162 \end{aligned}$$

4. **POOLS** Kenton's job is to cover the community pool during fall and winter. Since the pool is in the shape of an octagon, he needs to find the area in order to have a custom cover made. If the pool has the dimensions shown at the right, what is the area of the pool?



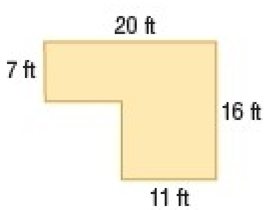
SOLUTION:

Since the polygon has 8 sides, the polygon can be divided into 8 congruent isosceles triangles, each with a base of 5 ft and a height of 6 ft.
Find the area of one triangle.

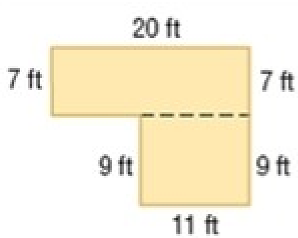
$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(5)(6)$$
$$= 15 \text{ ft}^2$$

Since there are 8 triangles, the area of the pool is $15 \cdot 8$ or 120 square feet.

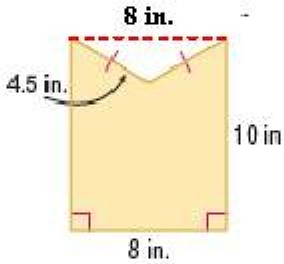
Find the area of each figure. Round to the nearest tenth if necessary.

5. 

SOLUTION:



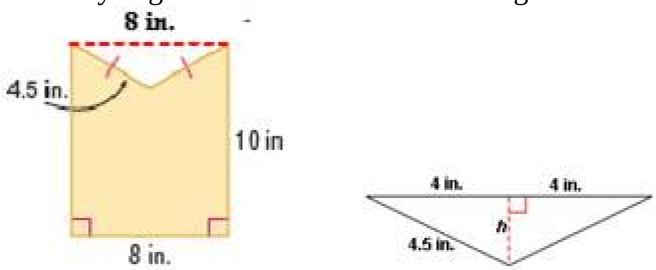
$$\begin{aligned}\text{Area of the figure given} &= 11(9) + 20(7)\\ &= 99 + 140\\ &= 239 \text{ ft}^2\end{aligned}$$

6. 

SOLUTION:

To find the area of the figure, subtract the area of the triangle from the area of the rectangle.

Use the Pythagorean Theorem to find the height *h* of the isosceles triangle at the top of the figure.



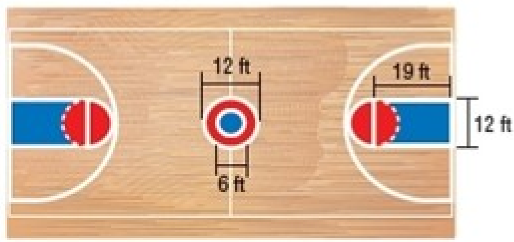
$$
\begin{aligned}
4^2 + h^2 &= 4.5^2 \\
h^2 &= 20.25 - 16 \\
h &= \sqrt{4.25}
\end{aligned}
$$

Area of the figure = Area of rectangle – Area of triangle

$$
\begin{aligned}
\text{Area of the figure} &= bh - \tfrac{1}{2}bh \\
&= 8(10) - \tfrac{1}{2}(8)\left(\sqrt{4.25}\right) \\
&\approx 80 - 8.2 \\
&\approx 71.8
\end{aligned}
$$

Therefore, the area of the figure is about 71.8 in^2.

7. **BASKETBALL** The basketball court in Jeff's school is painted as shown.



Note: Art not drawn to scale.

a. What area of the court is blue? Round to the nearest square foot.
b. What area of the court is red? Round to the nearest square foot.

SOLUTION:

a. The small blue circle in the middle of the floor has a diameter of 6 feet so its radius is 3 feet. The blue sections on each end are the area of a rectangle minus the area of half the red circle. The rectangle has dimensions of 12 ft by 19 ft. The diameter of the red circle is 12 feet so its radius is 6 feet.

Area of blue sections = Area of small blue circle + 2 [Area of rectangle - Area of red circle ÷ 2]

$$\begin{aligned} A(\text{blue}) &= \pi(3)^2 + 2\left[(12)(19) - \frac{\pi(6^2)}{2}\right] \\ &= 9\pi + 2[228 - 18\pi] \\ &\approx 28.27 + 342.90 \\ &\approx 371.17 \end{aligned}$$

So, the area of the court that is blue is about 371 ft^2.

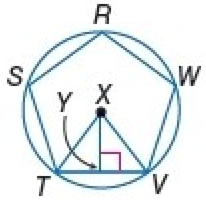
b. The two red circles on either end of the court each have a diameter of 12 feet or a radius of 6 feet. The large circle at the center of the court has a diameter of 12 feet so it has a radius of 6 feet. The inner blue circle has a diameter of 6 feet so it has a radius of 3 feet.

Area of red sections = 2 [Area of end red circles] - [Area of large center circle - Area of blue center circle]

$$\begin{aligned} A(\text{red}) &= 2\left[\pi(6)^2\right] + \left[\pi(6)^2 - \pi(3)^2\right] \\ &= 2[36\pi] + [36\pi - 9\pi] \\ &= 72\pi + 27\pi \\ &= 99\pi \\ &\approx 311.01 \end{aligned}$$

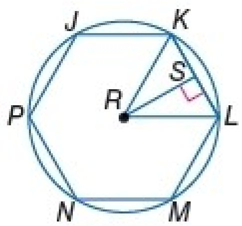
So, the area of the court that is red is about 311 ft^2.

In each figure, a regular polygon is inscribed in a circle. Identify the center, a radius, an apothem, and a central angle of each polygon. Then find the measure of a central angle.

8. 

SOLUTION:

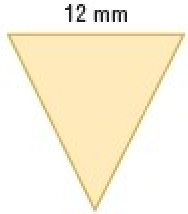
Center: point X, radius: $\overline{XV}$, apothem: $\overline{XY}$, central angle: $\angle VXT$, A square is a regular polygon with 4 sides. Thus, the measure of each central angle of square *RSTVW* is $\frac{360}{5}$ or 72.

9. 

SOLUTION:

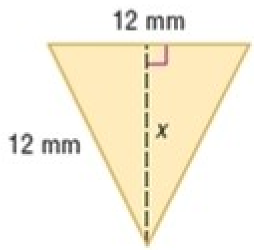
Center: point R, radius: $\overline{RL}$, apothem: $\overline{RS}$, central angle: $\angle KRL$, A square is a regular polygon with 4 sides. Thus, the measure of each central angle of square *JKLMNP* is $\frac{360}{6}$ or 60.

Find the area of each regular polygon. Round to the nearest tenth.

10. 

SOLUTION:

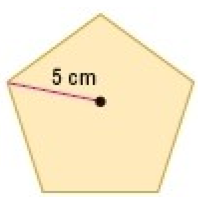
An equilateral triangle has three congruent sides. Draw an altitude and use the Pythagorean Theorem to find the height.



$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
6^2 + b^2 &= 12^2 \\
b^2 &= 12^2 - 6^2 \\
b^2 &= 144 - 36 \\
b &= \sqrt{108} \\
b &\approx 10.4
\end{aligned}
$$

Find the area of the triangle.

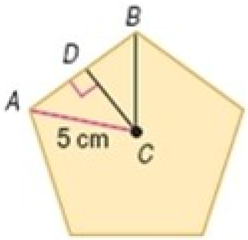
$$
\begin{aligned}
A &= \frac{1}{2}bh \\
&= \frac{1}{2}(12)\left(\sqrt{108}\right) \\
&\approx 62.4
\end{aligned}
$$

11. 

SOLUTION:

The formula for the area of a regular polygon is $A = \frac{1}{2}Pa$, so we need to determine the perimeter and the length of the apothem of the figure.

A regular pentagon has 5 congruent central angles, so the measure of central angle is $\frac{360}{5}$ or 72.



Apothem $\overline{DC}$ is the height of the isosceles triangle *ABC*. Triangles *ACD* and *BCD* are congruent, with $\angle ACD = \angle BCD = 36$.

Use the Trigonometric ratios to find the side length and apothem of the polygon.

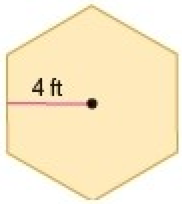
$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 36 = \frac{AD}{AC}$$
$$\sin 36 = \frac{AD}{5}$$
$$5\sin 36 = AD$$

$$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 36 = \frac{DC}{AC}$$
$$\cos 36 = \frac{DC}{5}$$
$$5\cos 36 = DC$$

$$AB = 2AD = 10\sin 36$$

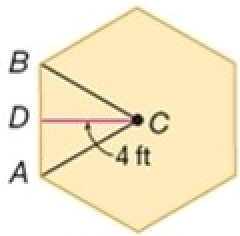
Use the formula for the area of a regular polygon.

$$\begin{aligned}\text{Area of the polygon} &= \frac{1}{2}aP \\ &= \frac{1}{2}(5\cos 36)(5 \times 10\sin 36) \\ &\approx 59.4 \text{ cm}^2\end{aligned}$$

12. 

SOLUTION:

A regular hexagon has 6 congruent central angles that are a part of 6 congruent triangles, so the measure of the central angle is $\frac{360}{6} = 60$.



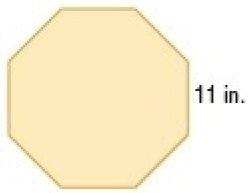
Apothem $\overline{DC}$ is the height of equilateral triangle *ABC* and it splits the triangle into two 30-60-90 triangles.

Use the Trigonometric ratio to find the side length of the polygon.

$$\tan\theta = \frac{opposite}{adjacent}$$
$$\tan 30 = \frac{AD}{DC}$$
$$\tan 30 = \frac{AD}{4}$$
$$4\tan 30 = AD$$

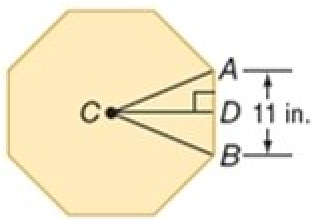
AB = 2(*AD*), so *AB* = 8 tan 30.

$$\begin{aligned}\text{Area of the polygon} &= \frac{1}{2}aP \\ &= \frac{1}{2}(4)(6 \times 8\tan 30) \\ &\approx 55.4 \text{ ft}^2\end{aligned}$$

13. 

SOLUTION:

A regular octagon has 8 congruent central angles, from 8 congruent triangles, so the measure of central angle is 360 ÷ 8 = 45.



Apothem $\overline{DC}$ is the height of the isosceles triangle *ABC* and it splits the triangle into two congruent triangles.

$\angle ACD = \angle BCD = 22.5$

Use the trigonometric ratio to find the apothem of the polygon.

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan 22.5 = \frac{AD}{DC}$$

$$DC = \frac{AD}{\tan 22.5}$$

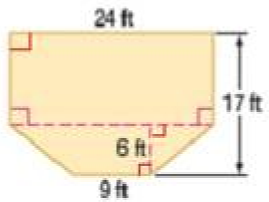
$$= \frac{5.5}{\tan 22.5}$$

$$\text{Area of the polygon} = \frac{1}{2}aP$$

$$= \frac{1}{2}\left(\frac{5.5}{\tan 22.5}\right)(8 \times 11)$$

$$\approx 584.2 \text{ in}^2$$

14. **CARPETING** Ignacio's family is getting new carpet in their family room, and they want to determine how much the project will cost.
a. Use the floor plan shown to find the area to be carpeted.
b. If the carpet costs $4.86 per square yard, how much will the project cost?



SOLUTION:

a. The longer dotted red line divides the floor into two quadrilaterals. The quadrilateral formed on top will have four right angles, so it is a rectangle with a base of 24 feet. The height of the rectangle is 17 – 6 = 11 feet.The longer dotted red side and the bottom side (9 ft side) are both perpendicular to the shorter dotted red side (6 ft side) so they are parallel to each other. Since the quadrilateral on the bottom has two parallel sides, it is a trapezoid with a height of 6 feet and bases of length 9 feet and 24 feet (opposite sides of a rectangle are congruent). The area of the room will be the sum of the area of the rectangle and the area of the trapezoid.

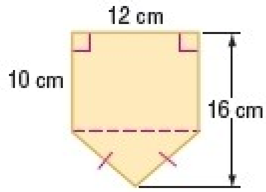
$$\begin{aligned} A(\text{room}) &= A(\text{rectangle}) + A(\text{trapezoid}) \\ &= bh + \tfrac{1}{2}h(b_1 + b_2) \\ &= (24)(11) + \tfrac{1}{2}(6)(24 + 9) \\ &= 264 + 99 \\ &= 363 \end{aligned}$$

So, the area of the floor to be carpeted is 363 ft^2.

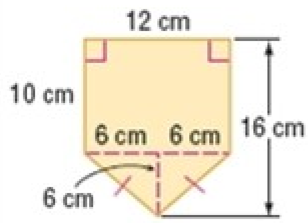
b. At $4.86 per yard, the project will cost:

$$\frac{\$4.86}{\text{yd}^2} \cdot \left(\frac{\text{yd}}{3\text{ft}}\right)^2 \cdot 363\,\text{ft}^2 = \$196.02$$

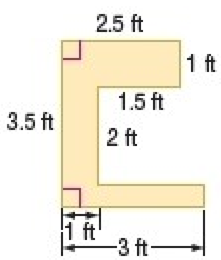
Find the area of each figure. Round to the nearest tenth if necessary.

15. 

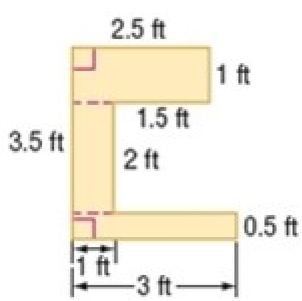
SOLUTION:



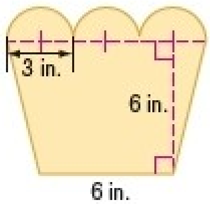
$$\begin{aligned}\text{Area} &= \text{rectangle} + \text{triangle}\\ &= 12(10) + \frac{1}{2} \times 12 \times 6\\ &= 120 + 36\\ &= 156 \text{ cm}^2\end{aligned}$$

16. 

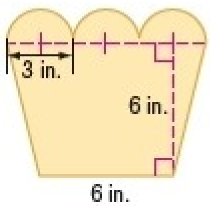
SOLUTION:



$$\begin{aligned}\text{Area} &= 2.5(1) + 2(1) + 3(0.5)\\ &= 6 \text{ ft}^2\end{aligned}$$

17. 

SOLUTION:

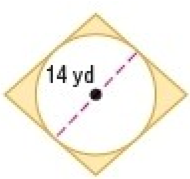


$$\begin{aligned}\text{Area} &= \text{trapezoid} + \text{triangle} + 3 \text{ semicircles}\\ &= \tfrac{1}{2}(7.5+6)6 + 3\left(\tfrac{1}{2}\pi(1.5)^2\right) + \tfrac{1}{2}(1.5)(6)\\ &\approx 40.5 + 10.6 + 4.5\\ &= 55.6 \text{ in}^2\end{aligned}$$

18. 

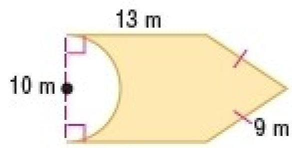
SOLUTION:

$$\begin{aligned}\text{Area} &= \text{sm. rectangle} + \text{lg rectangle} + \tfrac{1}{4}(\text{circle})\\ &= (3.5\times 3) + (5.5\times 3.5) + \tfrac{1}{4}\left(\pi(3.5)^2\right)\\ &\approx 10.5 + 19.25 + 9.62\\ &\approx 39.4 \text{ mm}^2\end{aligned}$$

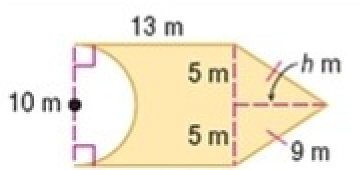
19. 

SOLUTION:

$$\begin{aligned}\text{Area} &= \text{square} - \text{circle}\\ &= (14\times 14) - \left(\pi(7)^2\right)\\ &= (14\times 14) - \left(\pi(7)^2\right)\\ &\approx 42.1 \text{yd}^2\end{aligned}$$

20. 

SOLUTION:

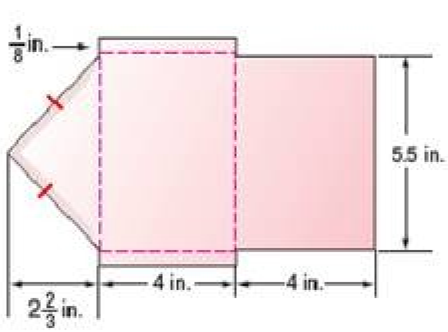


Use the Pythagorean Theorem to find *h*.

$$\begin{aligned} h^2 &= 9^2 - 5^2 \\ &= 81 - 25 \\ &= 56 \\ h &\approx 7.48 \end{aligned}$$

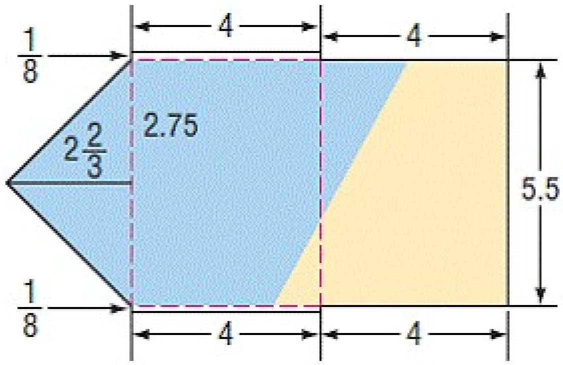
$$\begin{aligned} \text{Area} &= (\text{rectangle} - \text{ semi circle}) + \text{triangle} \\ &= \left(13 \times 10 - \frac{\pi(5)^2}{2}\right) + \left(\frac{1}{2} \times 10 \times 7.48\right) \\ &= (130 - 12.5\pi) + 37.4 \\ &\approx 128.1\text{m}^2 \end{aligned}$$

21. **CRAFTS** Latoya's greeting card company is making envelopes for a card from the pattern shown.
a. Find the perimeter and area of the pattern? Round to the nearest tenth.
b. If Latoya orders sheets of paper that are 2 feet by 4 feet, how many envelopes can she make per sheet?



SOLUTION:

a. To find the perimeter of the envelope, first use the Pythagorean theorem to find the missing sides of the isosceles triangle on the left.



The base of the isosceles triangle is 5.5 inches, so the height will bisect the base into two segments that each have a length of 2.75 inches.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ \left(2\tfrac{2}{3}\right)^2 + 2.75^2 &= c^2 \\ 7.11 + 7.56 &\approx c^2 \\ 14.67 &\approx c^2 \\ 3.83 &\approx c \end{aligned}$$

So, each side of the isosceles triangle is about 3.83 inches long.

Find the sum of the lengths of all the sides of the envelope pattern.

$$\begin{aligned} P &= \tfrac{1}{8} + 4 + \tfrac{1}{8} + 4 + 5.5 + 4 + \tfrac{1}{8} + 4 + \tfrac{1}{8} + 3.83 + 3.83 \\ &= 29..66 \end{aligned}$$

Thus, the perimeter of the pattern is about 29.7 inches.

The pattern can be divided into two rectangles and a triangle. The smaller rectangle is 5.5 inches by 4 inches. The large rectangle is 4 inches by 5.5 + $\frac{2}{8}$ or 5.75 inches. The triangle has a base of 5.5 inches and a height of $2\frac{2}{3}$ inches.

Area of pattern = Area of large rectangle) + Area of small rectangle) + Area of triangle

$$\begin{aligned} A(\text{pattern}) &= b_1h_1 + b_2h_2 + \tfrac{1}{2}b_1h_3 \\ &= (5.5)(4) + (4)(5.75) + \tfrac{1}{2}\left(5.5\right)\left(2\tfrac{2}{3}\right) \\ &\approx 22 + 23 + 7.3 \\ &\approx 52.3 \end{aligned}$$

Therefore, the area of the pattern is about 52.3 in^2.

b. Dividing the area of the sheet of paper by the area of the pattern will not give us the number of envelopes per sheet. This does not allow for the paper lost due to the shape of the pattern. The number of envelopes per sheet will be determined by how many of the pattern shapes will fit on the paper.

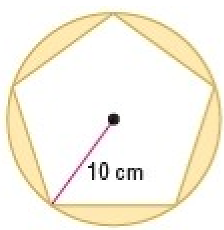
The maximum width of the pattern is $5.5+\frac{1}{8}+\frac{1}{8}$ or 5.75 inches. The sheet of paper has a width of 2 feet or 24 inches.
$24 \div 5.75 \approx 4.17$ so 4 patterns can be placed widthwise on the paper.

The maximum length of the pattern is $4+4+2\frac{2}{3}$ or $10\frac{2}{3}$ inches. The sheet of paper has a length of 4 feet or 48 inches.
$48 \div 10\frac{2}{3} = 4\frac{1}{2}$ so 4 patterns can be placed lengthwise on the paper.

Four patterns across by four patterns high will make a total of 4×4 or 16.

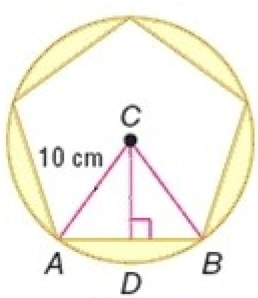
So, Latoya can make 16 cards per sheet.

Find the area of the shaded region formed by each circle and regular polygon. Round to the nearest tenth.

22. 

SOLUTION:

A regular pentagon has 5 congruent central angles, so the measure of central angle *ACB* is $\frac{360}{5}$ or 72.



Apothem $\overline{DC}$ is the height of the isosceles triangle *ABC*, so it bisects $\angle ACB$. Thus, $m\angle ACD = 36$. Use the trigonometric ratios to find the side length and apothem of the polygon.
Find *AD*.

$$\sin 36° = \frac{AD}{AC}$$
$$\sin 36° = \frac{AD}{10}$$
$$10\sin 36° = AD$$

Find DC.

$$\cos 36° = \frac{DC}{AC}$$
$$\cos 36° = \frac{DC}{10}$$
$$10\cos 36° = DC$$

ΔABC is an isosceles triangle, so $AB = 2(AD)$ or 20 sin 36°.

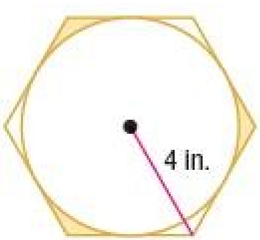
Use the formula for finding the area of a regular polygon replacing a with DC and p with $5(AB)$.

$$\begin{aligned}\text{Area of the pentagon} &= \tfrac{1}{2}aP\\ &= \tfrac{1}{2}(DC)(5 \times AB)\\ &= \tfrac{1}{2}(10\cos 36)(5 \times 20\sin 36)\\ &\approx 237.76 \text{ cm}^2\end{aligned}$$

Use the formula for the area of a circle replacing r with AC.

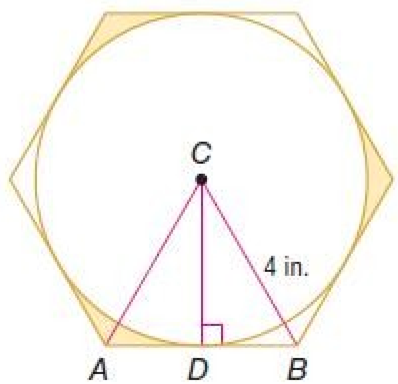
$$\begin{aligned}\text{Area of the circle} &= \pi r^2\\ &= \pi(AC)^2\\ &= \pi(10)^2\\ &= 100\pi \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of the shaded region} &= \text{Area of the circle} - \text{Area of the pentagon}\\ &= 100\pi - 237.76\\ &\approx 76.4 \text{ cm}^2\end{aligned}$$

23. 

SOLUTION:

First, find the area of the regular hexagon. A regular hexagon has 6 congruent central angles, so the measure of central angle ACB is $\frac{360}{6}$ or 60.



Since $AC = BC = 4$, $m\angle CAB = m\angle CBA$ and ΔABC is equilateral. Thus, $AB = BC = 4$ and the apothem $\overline{DC}$ is the height of an equilateral triangle ABC and bisects $\angle ACB$. So, $m\angle BCD = 30$. Use a trigonometric ratio to find the length of the $\overline{DC}$.

$$\cos 30° = \frac{DC}{BC}$$
$$\cos 30° = \frac{DC}{4}$$
$$4\cos 30° = DC$$

Use the formula for a regular polygon and replace a with DC and p with $6(AB)$ to find the area of the hexagon.

$$\begin{aligned}\text{Area of the hexagon} &= \frac{1}{2}aP \\ &= \frac{1}{2}(DC)(6 \times AB) \\ &= \frac{1}{2}(4\cos 30)(6 \times 4) \\ &\approx 41.57 \text{ ft}^2\end{aligned}$$

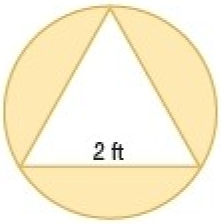
Find the area of the circle by replacing r in the area formula with DC.

$$\begin{aligned}\text{Area of the circle} &= \pi r^2 \\ &= \pi(DC)^2 \\ &= \pi(4\cos 30)^2 \\ &\approx 37.7 \text{ ft}^2\end{aligned}$$

Three of the six equal sections between the circle and the hexagon have been shaded, so the area of the shaded region is half the difference of the areas of the hexagon and the circle.

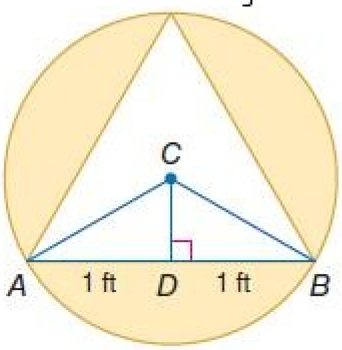
$$\begin{aligned}\text{Area of the shaded region} &= \frac{1}{2}(\text{Area of the circle} - \text{area of the hexagon}) \\ &= \frac{1}{2}(41.57 - 37.7) \\ &= \frac{1}{2}(3.87) \\ &\approx 1.9 \text{ in}^2\end{aligned}$$

The area of the shaded region is about 1.9 in^2.

24. 

SOLUTION:

First, find the area of the regular triangle. A regular triangle has 3 congruent central angles, so the measure of central angle ACB is $\frac{360}{3}$ or 120.



Since all radii for a circle are equal, $AC = BC$ and ΔABC is isosceles. The apothem $\overline{DC}$ is the height of ΔABC and will bisect $\overline{AB}$ and $\angle ACB$. Thus, $AD = 1$ and $m\angle ACD = 60$. Use trigonometric ratios to find the lengths of the $\overline{DC}$ and $\overline{AC}$.

$$\begin{aligned}\tan 60 &= \frac{AD}{DC}\\ \tan 60 &= \frac{1}{DC}\\ DC &= \frac{1}{\tan 60}\end{aligned}$$

$$\begin{aligned}\sin 60 &= \frac{AD}{AC}\\ \sin 60 &= \frac{1}{AC}\\ AC &= \frac{1}{\sin 60}\end{aligned}$$

Use the formula for a regular polygon and replace a with DC and p with $3(AB)$ to find the area of the triangle.

$$\begin{aligned}\text{Area of regular triangle} &= \tfrac{1}{2}ap \\ &= \tfrac{1}{2}(DC)(3\times AB) \\ &= \tfrac{1}{2}\left(\tfrac{1}{\tan 60}\right)(3\times 2) \\ &\approx 1.7\text{ ft}^2\end{aligned}$$

Find the area of the circle by replacing r in the area formula with AC.

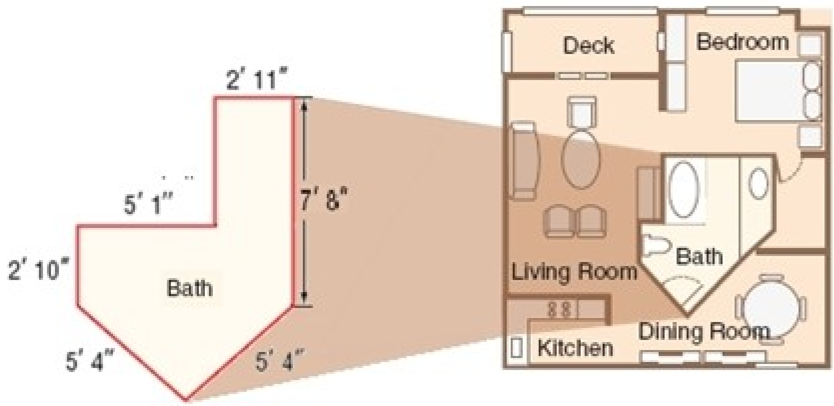
$$\begin{aligned}\text{Area of circle} &= \pi r^2 \\ &= \pi(AC)^2 \\ &= \pi\left(\tfrac{1}{\sin 60}\right)^2 \\ &\approx 4.2\text{ ft}^2\end{aligned}$$

The area of the shaded region is the difference of the areas of the circle and the triangle.

$$\begin{aligned}\text{Area of shaded region} &= \text{Area of circle} - \text{Area of triangle} \\ &= 4.2 - 1.7 \\ &= 2.5\text{ ft}^2\end{aligned}$$

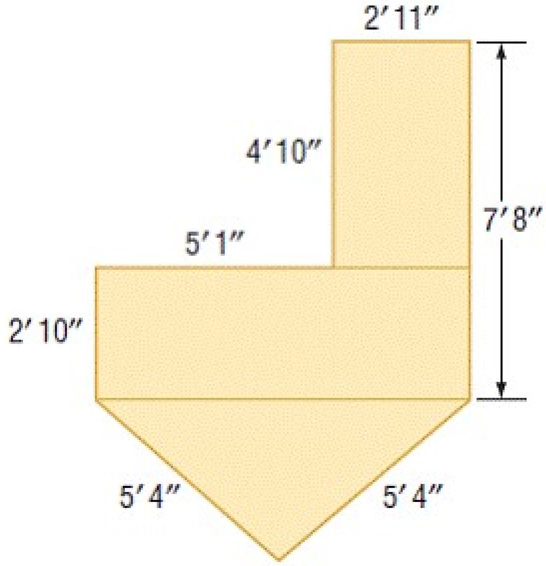
So, the area of the shaded region is about 2.5 ft^2.

25. **CONSTRUCTION** Find the area of the bathroom floor in the apartment floor plan.

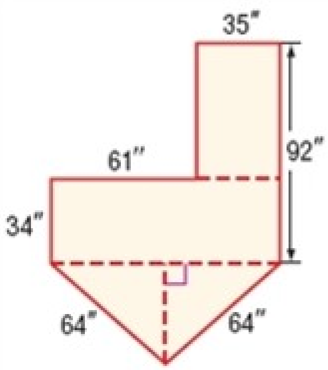


SOLUTION:

The total area of the bathroom floor is the sum of the areas of the vertical rectangle, the horizontal rectangle and the isosceles triangle shown.



Convert the given measures into inches and relabel the diagram.
2 ft 11 in. = 2(12) + 11 or 35 in.
7 ft 8 in. = 7(12) + 8 or 92 in.
5 ft 4 in. = 5(12) + 4 or 64 in.
2 ft 10 in. = 2(12) + 10 or 34 in.
5 ft 1 in. = 5(12) + 1 or 61 in.



The area of the vertical rectangle is 35(92 – 34) or 2030 in^2.
The area of the horizontal rectangle is (61 + 35)34 or 3264 in^2.

An altitude of the isosceles triangle drawn from it's vertex to its base bisects the base and forms two right triangles. If the base of the triangle is 61 + 35 or 96 in., then the length of the smaller leg of one of the right triangles is 0.5(96) or 48 inches. The length of the other leg, the height of the triangle, can be found using the Pythagorean Theorem.

$$\text{height of triangle} = \sqrt{64^2 - 48^2}$$
$$= \sqrt{1792}$$

The area of the triangle is $\frac{1}{2}(96)\left(\sqrt{1972}\right) \approx 2031.937\cdot$

The total area of the bathroom floor is about 2030 + 3264 + 2031.937 or 7325.937 in^2.

Convert to square feet.

$$7325.937 \cdot \frac{1\ \text{ft}^2}{144\ \text{in}^2} \approx 50.9$$

Find the perimeter and area of the figure. Round to the nearest tenth, if necessary.

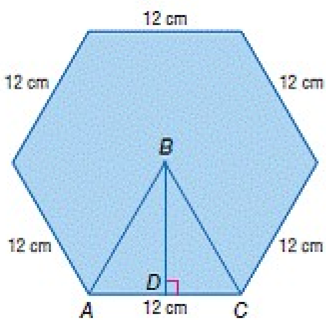
26. a regular hexagon with a side length of 12 centimeters

SOLUTION:

A regular hexagon has 6 equal side lengths, so the perimeter is

$$\begin{aligned} P &= n \cdot s \\ &= 6 \cdot 12 \\ &= 72 \end{aligned}$$

To find the area we first need to find the apothem.



$$\tan 30° = \frac{6}{\overline{BD}}$$

$$\overline{BD} = \frac{6}{\tan 30°}$$

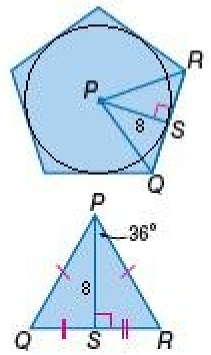
$$\overline{BD} \approx 10.39$$

The length of the apothem is about 10.39.

$$\begin{aligned} A &= \frac{1}{2}aP \\ &= \frac{1}{2}\left(\frac{6}{\tan 30°}\right)(72) \\ &= 374.1 \end{aligned}$$

27. a regular pentagon circumscribed about a circle with a radius of 8 millimeters

SOLUTION:



Use trigonometry to determine the side length of the pentagon.

$$\tan 36^\circ = \frac{\overline{SR}}{8}$$
$$\overline{SR} = 8\tan 36^\circ$$

$$\begin{aligned}\overline{QR} &= 2\overline{SR}\\ &= 2 \cdot 8\tan 36^\circ\\ &= 16\tan 36^\circ\\ &\approx 11.62\end{aligned}$$

$$\begin{aligned}P &= 5\overline{QR}\\ &= 5(16\tan 36^\circ)\\ &\approx 58.1\end{aligned}$$

$$\begin{aligned}A &= \frac{1}{2}aP\\ &= \frac{1}{2}(8)(58.1)\\ &\approx 232.4\end{aligned}$$

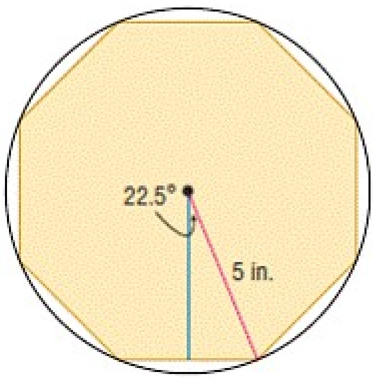
28. a regular octagon inscribed in a circle with a radius of 5 inches

SOLUTION:

A regular octagon has 8 congruent central angles, from 8 congruent triangles, so the measure of central angle is 360 ÷ 8 = 45.

The apothem splits the triangle into two congruent triangles, cutting the central angle in half.

The octagon is inscribed in a circle, so the radius of the circle is congruent to the radius of the octagon.



Use trigonometry to find the apothem and the length of each side of the octagon.

$$\begin{aligned} \sin x &= \frac{\text{opposite}}{\text{hypotenuse}} \\ \sin 22.5 &= \frac{x}{5} \\ 5\sin 22.5 &= x \end{aligned}$$

The length of each side is 10 sin 22.5.

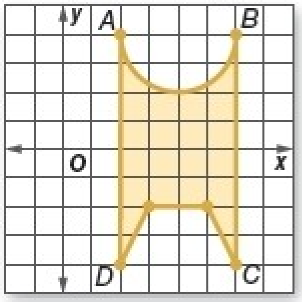
$$\begin{aligned} \cos x &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ \cos 22.5 &= \frac{y}{5} \\ 5\cos 22.5 &= y \end{aligned}$$

The length of the apothem is 5 cos 22.5.

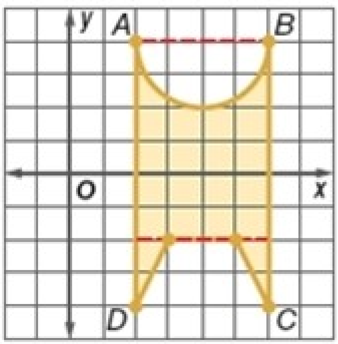
$$\begin{aligned} P &= 8(10\sin 22.5) \\ &= 80\sin 22.5 \\ &\approx 30.6 \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2}Pa \\ &= \frac{1}{2}(80\sin 22.5)(5\cos 22.5) \\ &\approx 70.7 \end{aligned}$$

Find the area of each shaded region. Round to the nearest tenth.

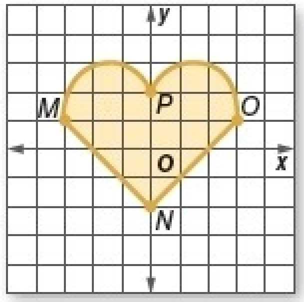
29. 

SOLUTION:

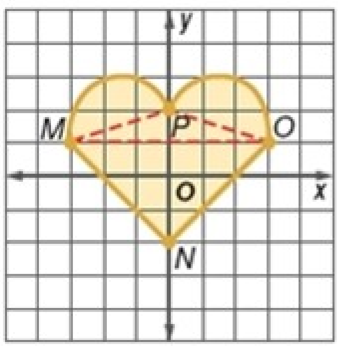


To find the area of the shaded region, subtract the area of the semicircle and the area of the trapezoid from the area of the rectangle.

$$\begin{aligned} A &= ABCD - \text{trapzoid} - \text{semicircle} \\ &= (4)(8) - \left[\tfrac{1}{2}(2+4)(2)\right] - \tfrac{1}{2}\pi(2)^2 \\ &= 32 - 6 - 2\pi \\ &= 26 - 2\pi \\ &\approx 19.7 \end{aligned}$$

30. 

SOLUTION:



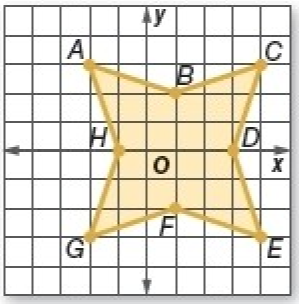
To find the area of the figure, separate it into triangle MNO with a base of 6 units and a height of 3 units, two semicircles, and triangle MPO with a base of 6 units and a height of 1 unit.

First, use the Distance Formula to find the diameter of one semicircle.

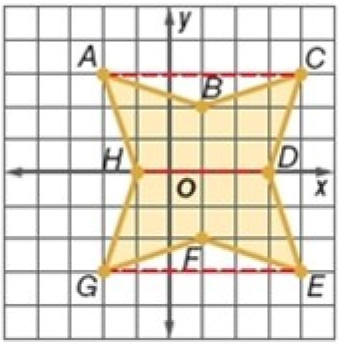
$$MP = \sqrt{(0+3)^2 + (2-1)^2}$$
$$= \sqrt{10}$$

So, the radius is $\frac{\sqrt{10}}{2}$.

$$\text{Area} = \Delta MNO + 2(\text{semicircle}) + \Delta MPO$$
$$= \frac{1}{2}bh + 2\left(\frac{180}{360}\pi r^2\right) + \frac{1}{2}bh$$
$$= \frac{1}{2}(6)(3) + 2\left[\frac{180}{360}\pi\left(\frac{\sqrt{10}}{2}\right)^2\right] + \frac{1}{2}(6)(1)$$
$$= 9 + 2.5\pi + 3$$
$$\approx 19.9$$

31. 

SOLUTION:



Using *DH* as a divider, we have two trapezoids, *ACDH* and *GEDH*. We need to find the areas of these and subtract the areas of the two triangles, *ABC* and *GFE*.

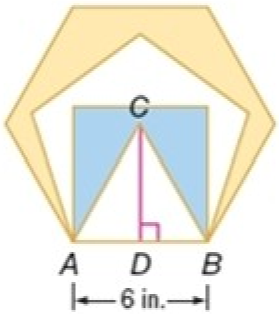
$$\begin{aligned} A &= ACDH - ABC + (GEDH - GFE) \\ &= \tfrac{1}{2}(6+4)3 - \tfrac{1}{2}(6)(1) + \left[\tfrac{1}{2}(6+4)3 - \tfrac{1}{2}(6)(1)\right] \\ &= 15 - 3 + (15 - 3) \\ &= 12 + 12 \\ &= 24 \end{aligned}$$

32. Find the total area of the shaded regions. Round to the nearest tenth.

SOLUTION:

Find the area of the hexagon.

A regular hexagon has 6 congruent triangles with 6 congruent central angles, so the measure of one central angle is 360 ÷ 6 = 60.



Apothem $\overline{DC}$ is the height of an equilateral triangle ABC. Use the trigonometric ratios to find the apothem of the polygon.

$$\tan 30° = \frac{AD}{CD}$$
$$\tan 30° = \frac{3}{CD}$$
$$CD \tan 30° = 3$$
$$CD = \frac{3}{\tan 30°}$$

$$\begin{aligned}\text{Area(hexagon)} &= \frac{1}{2}aP \\ &= \frac{1}{2}\left(\frac{3}{\tan 30°}\right)(6 \times 6) \\ &\approx 93.53 \text{ in}^2\end{aligned}$$

A regular pentagon has 5 congruent triangles with 5 congruent central angles, so the measure of each central angle is 360 ÷ 5 = 72.

Find the area of a regular pentagon with a side length of 6 inches.

$$\begin{aligned}\text{Area(pentagon)} &= \frac{1}{2}aP \\ &= \frac{1}{2}\left(\frac{3}{\tan 36°}\right)(5 \times 6) \\ &\approx 61.937 \text{ in}^2\end{aligned}$$

$$\begin{aligned}\text{Area(yellow)} &= \text{area(hexagon)} - \text{area(pentagon)} \\ &= 93.53 - 61.937 \\ &= 31.593\end{aligned}$$

$$\begin{aligned}\text{Area(blue)} &= \text{area(square)} - \text{area(triangle)} \\ &= 6(6) - 25.588 \\ &= 20.412\end{aligned}$$

$$\begin{aligned}\text{Area(total)} &= \text{area(yellow)} + \text{area(blue)} \\ &= 31.593 + 20.412 \\ &= 52.005\end{aligned}$$

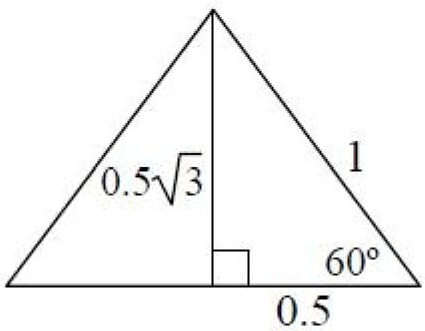
The area of the shaded region is about 52 in^2.

33. **CHANGING DIMENSIONS** Calculate the area of an equilateral triangle with a perimeter of 3 inches. Calculate the areas of a square, a regular pentagon, and a regular hexagon with perimeters of 3 inches. How does the area of a regular polygon with a fixed perimeter change as the number of sides increases?

SOLUTION:

Equilateral Triangle

The perimeter of an equilateral triangle is 3 inches, so the length of each side of the triangle is 1 inch.



$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(1)\left(0.5\sqrt{3}\right) \\ &\approx 0.43 \end{aligned}$$

Square

The perimeter of the square is 3 inches, so the length of each side of the square is 0.75 inch.

$$\begin{aligned} A &= s^2 \\ &= (0.75)^2 \\ &\approx 0.56 \end{aligned}$$

Regular pentagon

The perimeter of the regular pentagon is 3 inches, the length of each side of the pentagon is 0.6 inch.

$$\begin{aligned} A &= \frac{1}{2}aP \\ &= \frac{1}{2}\left(\frac{0.3}{\tan 36°}\right)(5 \times 0.6) \\ &\approx 0.62 \end{aligned}$$

Regular hexagon

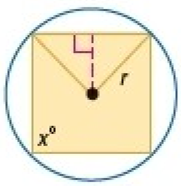
The perimeter of the regular hexagon is 3 inches, the length of each side of the pentagon is 0.5 inch.

$$A = \frac{1}{2}aP$$
$$= \frac{1}{2}\left(\frac{0.25}{\tan 30°}\right)(6 \times 0.5)$$
$$\approx 0.65$$

Sample answer: When the perimeter of a regular polygon is constant, as the number of sides increases, the area of the polygon increases.

34. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the areas of regular polygons inscribed in circles.

a. GEOMETRIC Draw a circle with a radius of 1 unit and inscribe a square. Repeat twice, inscribing a regular pentagon and hexagon.



b. ALGEBRAIC Use the inscribed regular polygons from part **a** to develop a formula for the area of an inscribed regular polygon in terms of angle measure *x* and number of sides *n*.

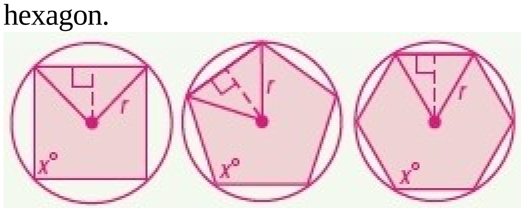
c. TABULAR Use the formula you developed in part **b** to complete the table below. Round to the nearest hundredth.

Number of sides, n	4	5	6	8	10	20	50	100
Interior Angle Measure, x								
Area of Inscribed Regular Polygon								

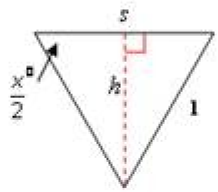
d. VERBAL Make a conjecture about the area of an inscribed regular polygon with a radius of 1 unit as the number of sides increases.

SOLUTION:

a.Use a compass to construct a circle with a radius of 1 unit. Use a protractor to draw a 90° central angle. Set the compass for the width of the two points of intersection of the circle and the angle. Use the compass to mark off two more points on the circle at that same width. Connect the four points to construct the inscribed square. Construct another circle and draw a 72° central angle. Mark off 3 more points using the width of the points of intersection and connect to form an inscribed regular pentagon. Then construct a third circle and draw a 60° angle. Mark off 4 additional points using the width of the points of intersection. Connect the points to construct an inscribed regular hexagon.



b. The area of each inscribed regular polygon of *n* sides is *n* times the area of the isosceles triangle with legs of 1 unit created by the central angle that was drawn. Since all *n* triangles are congruent, the base angles of the triangle are each half of the interior angle of the regular polygon. So, $x = \frac{180(n-2)}{n}$ for each regular polygon and the measure of the base angle is $\frac{x}{2}$.



Use trigonometric ratios to find expressions for the height h and base s of the triangle in terms of x and then write an expression for the area of the triangle.

$$\begin{aligned} \sin\left(\frac{x}{2}\right) &= \frac{h}{1} \\ \sin\left(\frac{x}{2}\right) &= h \end{aligned}$$

$$\begin{aligned} \cos\left(\frac{x}{2}\right) &= \frac{\frac{s}{2}}{1} \\ \cos\left(\frac{x}{2}\right) &= \frac{s}{2} \\ 2\cos\left(\frac{x}{2}\right) &= s \end{aligned}$$

$$\begin{aligned} A(\text{triangle}) &= \frac{1}{2}bh \\ &= \frac{1}{2}\left[2\cos\left(\frac{x}{2}\right)\right]\left[\sin\left(\frac{x}{2}\right)\right] \\ &= \cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right) \end{aligned}$$

For each inscribed regular polygon of n sides, there are n congruent isosceles triangles. Multiply to find the area of the regular polygon.

$$\begin{aligned} A(\text{regular polygon}) &= n \cdot A(\text{triangle}) \\ &= n\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right) \end{aligned}$$

c. To find the area of each inscribed regular polygon, first find the measure of its interior angles.

For $n = 4$:
$x = \frac{180(4-2)}{4}$ or $90°$
$A_4 = 4\cos\left(\frac{90}{2}\right)\sin\left(\frac{90}{2}\right)$ or $4\cos 45 \sin 45$

For $n = 5$:
$x = \frac{180(5-2)}{5}$ or $108°$
$A_5 = 5\cos\left(\frac{108}{2}\right)\sin\left(\frac{108}{2}\right)$ or $5\cos 54 \sin 54$

For $n = 6$:

$x = \frac{180(6-2)}{6}$ or $120°$

$A_6 = 6\cos\left(\frac{120}{2}\right)\sin\left(\frac{120}{2}\right)$ or $6\cos 60 \sin 60$

For $n = 8$:

$x = \frac{180(8-2)}{8}$ or $135°$

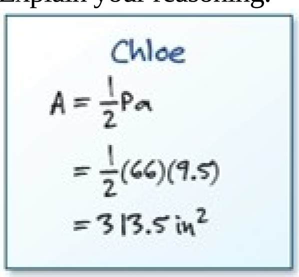
$A_8 = 8\cos\left(\frac{135}{2}\right)\sin\left(\frac{135}{2}\right)$ or $8\cos 67.5 \sin 67.5$

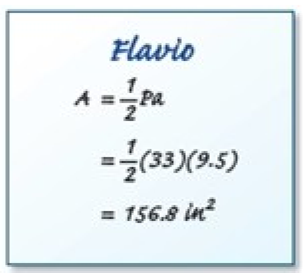
Number of Sides, n	4	5	6	8	10	20	50	100
Interior Angle Measure, x	90°	108°	120°	135°	144°	162°	172.8°	176.4°
Area of Inscribed Regular Polygon	2.00	2.38	2.60	2.83	2.94	3.09	3.13	3.14

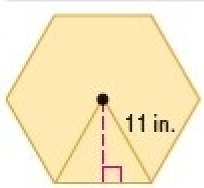
d.

Sample answer: The area of a circle with radius 1 is $A = \pi(1)^2$ or about 3.1416. As the number of sides of the polygon increases, the area of a polygon inscribed in a circle approaches the area of the circle or π.

35. **ERROR ANALYSIS** Chloe and Flavio want to find the area of the hexagon shown. Is either of them correct? Explain your reasoning.







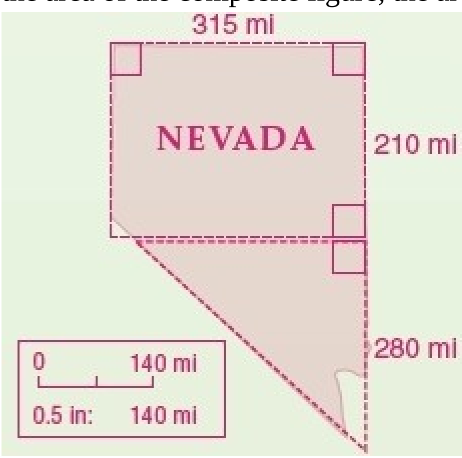
SOLUTION:

Chloe; sample answer: The measure of each angle of a regular hexagon is 120°, so the segments from the center to each vertex form 60° angles. The triangles formed by the segments from the center to each vertex are equilateral, so each side of the hexagon is 11 in. The perimeter of the hexagon is 66 in. Using trigonometry, the length of the apothem is about 9.5 in. Putting the values into the formula for the area of a regular polygon and simplifying, the area is about $313.5\ in^2$.

36. **CHALLENGE** Using the map of Nevada shown, estimate the area of the state. Explain your reasoning.

SOLUTION:

Sample answer: Divide Nevada into a rectangle that is about 315 miles by about 210 miles and a right triangle with a base of about 315 miles and a height of about 280 miles. Finding the areas of the two basic figures and adding to find the area of the composite figure, the area of Nevada is about $110,250 \text{ mi}^2$.



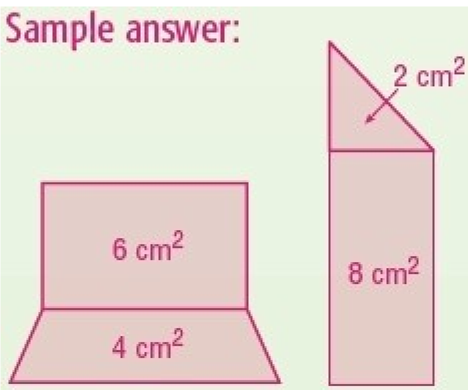
37. **OPEN ENDED** Draw a pair of composite figures that have the same area. Make one composite figure out of a rectangle and a trapezoid, and make the other composite figure out of a triangle and a rectangle. Show the area of each basic figure.

SOLUTION:

Have the areas of the figures each sum to a basic value, like 10 cm^2.

Set the first rectangle equal to 6 cm^2 with a base of 3 cm and a height of 2 cm. Set the trapezoid below the rectangle, so the top base must be 3 cm. If the height of the trapezoid is 1 cm, then the bottom base must be 5 cm, so the area of the trapezoid is 0.5(1)(3 +5) = 4 cm^2.

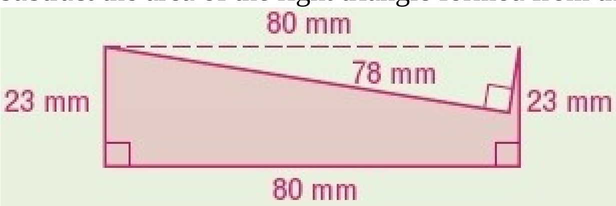
For the second figure, set the triangle to be a base and height of 2 cm, with an area of 2 cm^2. The rectangle should connect to the base of the triangle and by 2 cm by 4 cm to have an area of 8 cm^2.



38. **REASONING** Is enough information provided to find the area of the shaded figure? If so, what is the area to the nearest square millimeter? If not, what additional information is needed? Explain.

SOLUTION:

Yes; sample answer: You can draw a horizontal line across the top of the figure to complete the rectangle. Then subtract the area of the right triangle formed from the area of the rectangle to find the area of the figure.



Use the Pythagorean Theorem to find the missing side length of the triangle.

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
78^2 + b^2 &= 80^2 \\
b^2 &= 80^2 - 78^2 \\
b^2 &= 6400 - 6084 \\
b^2 &= 316 \\
b &\approx 17.78
\end{aligned}
$$

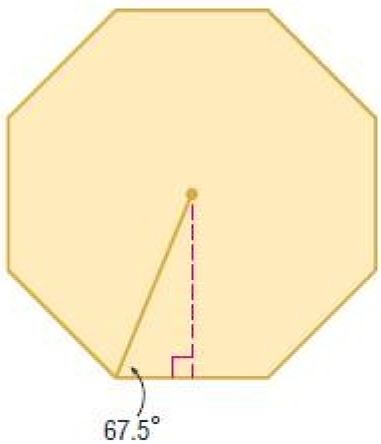
$$
\begin{aligned}
\text{Area of the shaded region} &= \text{area of a rectangle} - \text{area of a triangle} \\
&= \ell w - \frac{1}{2}bh \\
&= (80)(23) - \frac{1}{2}(17.78)(78) \\
&= 1840 - 693.42 \\
&\approx 1146.58
\end{aligned}
$$

So, the area of the shaded region is about 1147 mm^2.

39. **WRITING IN MATH** Explain how to find the area of a regular octagon if you are only given the distance from the center of the figure to a vertex.

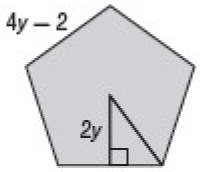
SOLUTION:

Sample answer: If you draw an apothem to the side of the octagon to form a right triangle with the side of the figure and the segment from the center to the vertex, you can use the triangle to find the apothem and the perimeter. Since the octagon is a regular octagon, the measure of each vertex is 135°, so the angle formed by the side of the figure and the segment from the center to the vertex is 67.5°.



Using the sine ratio, the length of the apothem is the radius multiplied by sin 67.5. Using the cosine ratio, one half of the length of a side is the radius multiplied by cos67.5, so the side is twice the radius multiplied by cos67.5. Find the perimeter by multiplying the length of the side by 8. Use the formula $A = \frac{1}{2}pa$ with the perimeter and apothem lengths you calculated.

40. Which polynomial best represents the area of the regular pentagon shown below?



A $10y^2 - 5$

B $10y^2 + 5y$

C $20y^2 + 10$

D $20y^2 - 10y$

SOLUTION:

The given regular pentagon can be divided into five congruent triangles.

The area of each triangle is $\frac{1}{2}bh$.

$$\begin{aligned} A &= 5\left[\tfrac{1}{2}bh\right] \\ &= 5\left[\tfrac{1}{2}(4y-2)(2y)\right] \\ &= 5\left[4y^2 - 2y\right] \\ &= 20y^2 - 10y \end{aligned}$$

41. What is $27^{-\frac{2}{3}}$ in radical form?

F $\frac{1}{\left(\sqrt[3]{27}\right)^2}$

G $\frac{1}{\left(\sqrt{27}\right)^2}$

H $\left(\sqrt[3]{27}\right)^2$

J $\left(\sqrt{27}\right)^3$

SOLUTION:

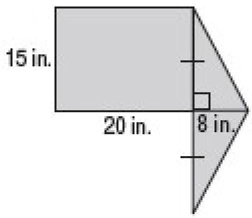
$$27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}}$$

$$= \frac{1}{\left(27^{\frac{1}{3}}\right)^2}$$

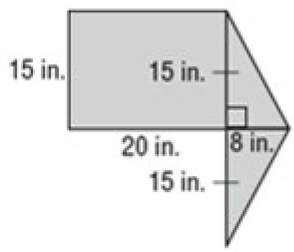
$$= \frac{1}{\left(\sqrt[3]{27}\right)^2}$$

So, the correct choice is F.

42. **SHORT RESPONSE** Find the area of the shaded figure in square inches. Round to the nearest tenth.



SOLUTION:



$$\begin{aligned}\text{Area of the rectangle} &= \ell w\\ &= (20)(15)\\ &= 300\end{aligned}$$

$$\begin{aligned}\text{Area of one triangle} &= \frac{1}{2}bh\\ &= \frac{1}{2}(15)(8)\\ &= 60\end{aligned}$$

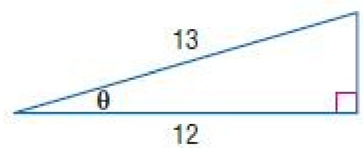
So, the area of the shaded region is 300 + 2(60) or 420 in^2.

43. **SAT/ACT** If the $\cos\theta = \frac{12}{13}$, what is the value of tan θ?

A $\frac{13}{5}$
B $\frac{12}{5}$
C $\frac{13}{12}$
D $\frac{5}{12}$
E $\frac{5}{13}$

SOLUTION:

Draw a right triangle to illustrate the problem using $\text{cosine} = \frac{\text{length of side adjacent to angle}}{\text{length of the hypotenuse}}$.



Find the length of the side opposite angle θ using the Pythagorean Theorem.

$$\begin{aligned}\text{length of side opposite }\theta &= \sqrt{13^2 - 12^2}\\ &= \sqrt{169 - 144}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

Use the definition to find the value of the tangent.

$$\tan\theta = \frac{\text{length of side opposite }\theta}{\text{length of side adjacent }\theta}$$

$$\tan\theta = \frac{5}{12}$$

Therefore, the correct choice is D.

Find the indicated measure. Round to the nearest tenth.

44. The area of a circle is 95 square feet. Find the radius.

SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ 95 &= \pi r^2 \\ \frac{95}{\pi} &= r^2 \\ \sqrt{\frac{95}{\pi}} &= r \\ 5.5 &\approx r \end{aligned}$$

45. Find the area of a circle whose radius is 9 centimeters.

SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(9)^2 \\ &= 81\pi \\ &\approx 254.5 \end{aligned}$$

46. The area of a circle is 256 square inches. Find the diameter.

SOLUTION:

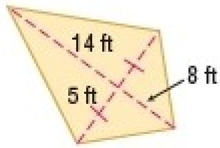
$$\begin{aligned} A &= \pi r^2 \\ 256 &= \pi r^2 \\ \frac{256}{\pi} &= r^2 \\ \sqrt{\frac{256}{\pi}} &= r \\ 2\sqrt{\frac{256}{\pi}} &= d \\ 18.1 &\approx d \end{aligned}$$

47. Find the area of a circle whose diameter is 25 millimeters.

SOLUTION:

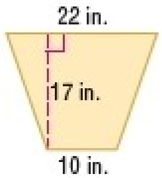
$$\begin{aligned} A &= \pi r^2 \\ &= \pi\left(\frac{25}{2}\right)^2 \\ &= 12.5^2\pi \\ &\approx 490.9 \end{aligned}$$

Find the area of each trapezoid, rhombus, or kite.

48. 

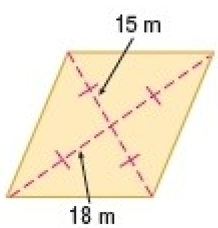
SOLUTION:

$$A = \frac{1}{2}d_1d_2$$
$$= \frac{1}{2}(10)(22)$$
$$= 110$$

49. 

SOLUTION:

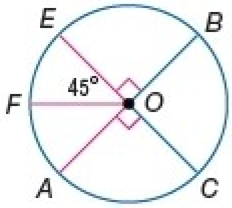
$$A = \frac{1}{2}(b_1 + b_2)h$$
$$A = \frac{1}{2}(10 + 22)(17)$$
$$A = 16 \cdot 17$$
$$A = 272$$

50. 

SOLUTION:

$$A = \frac{1}{2}d_1d_2$$
$$A = \frac{1}{2}(30)(36)$$
$$A = 540$$

$\overline{EC}$ and $\overline{AB}$ are diameters of $\odot O$. Identify each arc as a *major arc, minor arc,* or *semicircle* of the circle. Then find its measure.



51. $m\widehat{ACB}$

SOLUTION:

$\widehat{ACB}$ is the corresponding arc for $\overline{AB}$, which is a diameter.

Therefore, $\widehat{ACB}$ is a semicircle and $m\widehat{ACB} = 180.$

52. $m\widehat{EB}$

SOLUTION:

$\widehat{EB}$ is the shortest arc connecting the points *E* and *B* on $\odot O.$ It is a minor arc.

$m\angle EOB = 90.$

$m\widehat{EB} = 90.$

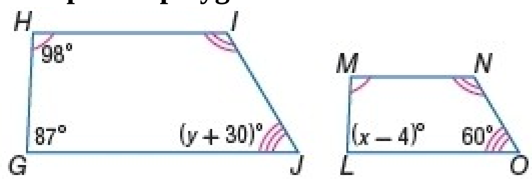
53. $m\widehat{ACE}$

SOLUTION:

$\widehat{ACE}$ is the longest arc connecting the points *A*, and *E* on $\odot O.$ It is a major arc.

$m\widehat{EB} = 90$ and $m\widehat{ACB} = 180.$
$m\widehat{ACE} = 90 + 180 = 270.$

Each pair of polygons is similar. Find *x*.

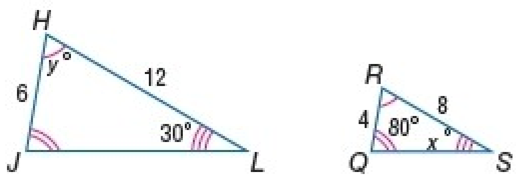
54. 

SOLUTION:

$\angle G$ corresponds with $\angle L$

$(x - 4)$ corresponds with 87°

$x = 91$

55. 

SOLUTION:

$\angle S$ corresponds with $\angle L$

x corresponds with 30°

$x = 30$