

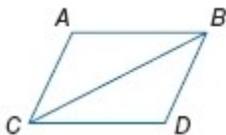
## 4-5 Proving Triangles Congruent - ASA, AAS

**PROOF** Write the specified type of proof.

1. two-column proof

Given:  $\overline{CB}$  bisects  $\angle ABD$  and  $\angle ACD$ .

Prove:  $\triangle ABC \cong \triangle DBC$



**SOLUTION:**

Proof:

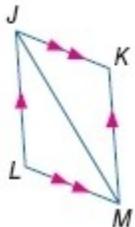
Statements (Reasons)

1.  $\overline{CB}$  bisects  $\angle ABD$  and  $\angle ACD$ . (Given)
2.  $\angle ABC \cong \angle DBC$  (Definition of angular bisector)
3.  $\overline{BC} \cong \overline{BC}$  (Reflexive Property)
4.  $\angle ACB \cong \angle DCB$  (Definition of angular bisector)
5.  $\triangle ABC \cong \triangle DBC$  (ASA)

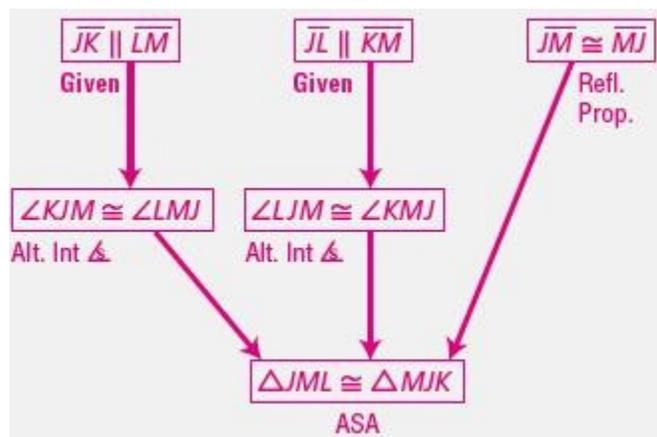
2. flow proof

Given:  $\overline{JK} \parallel \overline{LM}$ ,  $\overline{JL} \parallel \overline{KM}$

Prove:  $\triangle JML \cong \triangle MJK$



**SOLUTION:**

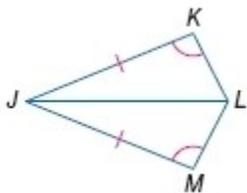


## 4-5 Proving Triangles Congruent - ASA, AAS

3. paragraph proof

Given:  $\angle K \cong \angle M$ ,  $\overline{JK} \cong \overline{JM}$ ,  $\overline{JL}$  bisects  $\angle KLM$ .

Prove:  $\triangle JKL \cong \triangle JML$



**SOLUTION:**

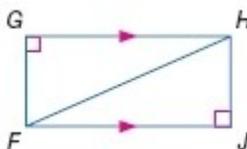
Proof: We are given  $\angle K \cong \angle M$ ,  $\overline{JK} \cong \overline{JM}$ , and  $\overline{JL}$  bisects  $\angle KLM$ . Since  $\overline{JL}$  bisects  $\angle KLM$ , we know  $\angle KLJ \cong \angle MLJ$ . So,  $\triangle JKL \cong \triangle JML$  by the Angle-Angle-Side Congruence Theorem.

4. two-column proof

Given:  $\overline{GH} \parallel \overline{FJ}$

$m\angle G = m\angle J = 90$

Prove:  $\triangle HJF \cong \triangle FGH$



**SOLUTION:**

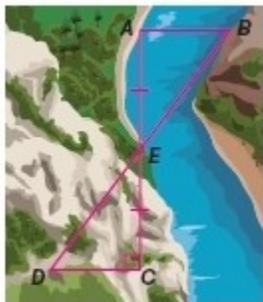
Proof:

Statements (Reasons)

1.  $\overline{GH} \parallel \overline{FJ}$ ,  $m\angle G = m\angle J = 90$  (Given)
2.  $\angle G \cong \angle J$  (Definition of congruent angles)
3.  $\angle GHF \cong \angle JFH$  (Alternate Interior angles are congruent.)
4.  $\overline{HF} \cong \overline{FH}$  (Reflexive Prop.)
5.  $\triangle HJF \cong \triangle FGH$  (AAS)

## 4-5 Proving Triangles Congruent - ASA, AAS

5. **BRIDGE BUILDING** A surveyor needs to find the distance from point  $A$  to point  $B$  across a canyon. She places a stake at  $A$ , and a coworker places a stake at  $B$  on the other side of the canyon. The surveyor then locates  $C$  on the same side of the canyon as  $A$  such that  $\overline{CA} \perp \overline{AB}$ . A fourth stake is placed at  $E$ , the midpoint of  $\overline{CA}$ . Finally, a stake is placed at  $D$  such that  $\overline{CD} \perp \overline{CA}$  and  $D$ ,  $E$ , and  $B$  are sited as lying along the same line.
- Explain how the surveyor can use the triangles formed to find  $AB$ .
  - If  $AC = 1300$  meters,  $DC = 550$  meters, and  $DE = 851.5$  meters, what is  $AB$ ? Explain your reasoning.

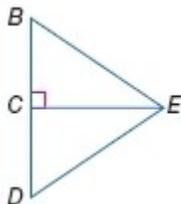


**SOLUTION:**

- We know  $\angle BAE$  and  $\angle DCE$  are congruent because they are both right angles.  $\overline{AE}$  is congruent to  $\overline{EC}$  by the Midpoint Theorem. From the Vertical Angles Theorem,  $\angle DEC \cong \angle BEA$ . By ASA, the surveyor knows that  $\triangle DCE \cong \triangle BAE$ . By CPCTC,  $\overline{DC} \cong \overline{AB}$ , so the surveyor can measure  $\overline{DC}$  and know the distance between  $A$  and  $B$ .
- Since  $DC = 550$  m and  $\overline{DC} \cong \overline{AB}$ , then by the definition of congruence,  $AB = 550$  m.

**PROOF** Write a paragraph proof.

6. **Given:**  $\overline{CE}$  bisects  $\angle BED$ ;  $\angle BCE$  and  $\angle ECD$  are right angles.  
**Prove:**  $\triangle ECB \cong \triangle ECD$



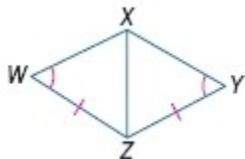
**SOLUTION:**

Proof: We are given that  $\overline{CE}$  bisects  $\angle BED$  and  $\angle BCE$  and  $\angle ECD$  are right angles. Since all right angles are congruent,  $\angle BCE \cong \angle ECD$ . By the definition of angle bisector,  $\angle BEC \cong \angle DEC$ . The Reflexive Property tells us that  $\overline{EC} \cong \overline{EC}$ .  
By Angle-Side-Angle Congruence Postulate,  $\triangle ECB \cong \triangle ECD$ .

## 4-5 Proving Triangles Congruent - ASA, AAS

7. **Given:**  $\angle W \cong \angle Y$ ,  $\overline{WZ} \cong \overline{YZ}$ ,  $\overline{XZ}$  bisects  $\angle WZY$ .

**Prove:**  $\triangle XWZ \cong \triangle XYZ$

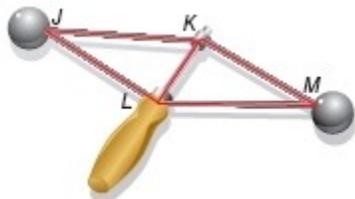


**SOLUTION:**

**Proof:** It is given that  $\angle W \cong \angle Y$ ,  $\overline{WZ} \cong \overline{YZ}$ , and  $\overline{XZ}$  bisects  $\angle WZY$ . By the definition of angle bisector,  $\angle WZX \cong \angle YZX$ .

The Angle-Side-Angle Congruence Postulate tells us that  $\triangle XWZ \cong \triangle XYZ$ .

8. **TOYS** The object of the toy shown is to make the two spheres meet and strike each other repeatedly on one side of the wand and then again on the other side. If  $\angle JKL \cong \angle MLK$  and  $\angle JLK \cong \angle MKL$ , prove that  $\overline{JK} \cong \overline{ML}$ .



**SOLUTION:**

**Proof:**

Statements (Reasons)

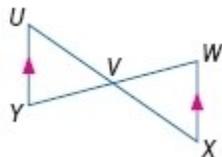
- $\angle JKL \cong \angle MLK$ ,  $\angle JLK \cong \angle MKL$  (Given)
- $\overline{KL} \cong \overline{KL}$  (Reflexive Prop.)
- $\triangle KLM \cong \triangle LKJ$  (ASA)
- $\overline{JK} \cong \overline{ML}$  (CPCTC)

**PROOF** Write a two-column proof.

9. **Given:** V is the midpoint of  $\overline{YW}$ ;

$$\overline{UY} \parallel \overline{XW}.$$

**Prove:**  $\triangle UVY \cong \triangle XVW$



**SOLUTION:**

**Proof:**

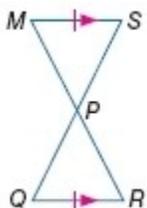
Statements (Reasons)

- V is the midpoint of  $\overline{YW}$ ;  $\overline{UY} \parallel \overline{XW}$ . (Given)
- $\overline{YV} \cong \overline{VW}$  (Midpoint Theorem)
- $\angle VWX \cong \angle VYU$  (Alternate Interior Angles Theorem)
- $\angle VUY \cong \angle VXW$  (Alternate Interior Angles Theorem)
- $\triangle UVY \cong \triangle XVW$  (AAS)

## 4-5 Proving Triangles Congruent - ASA, AAS

10. **Given:**  $\overline{MS} \cong \overline{RQ}, \overline{MS} \parallel \overline{RQ}$

**Prove:**  $\triangle MSP \cong \triangle RQP$



**SOLUTION:**

Proof:

Statements (Reasons)

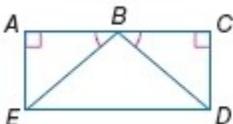
1.  $\overline{MS} \cong \overline{RQ}, \overline{MS} \parallel \overline{RQ}$  (Given)
2.  $\angle SPM \cong \angle QPR$  (Vertical angles are congruent.)
3.  $\angle SMP \cong \angle QRP$  (Alternate Interior Angle Theorem)
4.  $\triangle MSP \cong \triangle RQP$  (AAS)

11. **PROOF** Write a flow proof.

**Given:**  $\angle A$  and  $\angle C$  are right angles.

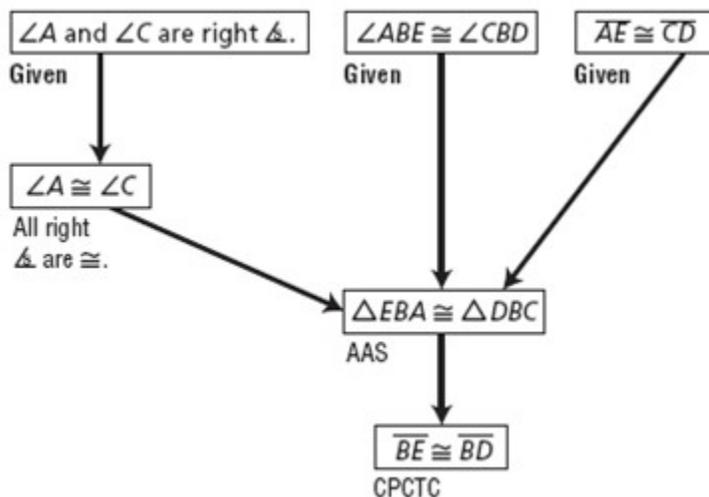
$\angle ABE \cong \angle CBD, \overline{AE} \cong \overline{CD}$

**Prove:**  $\overline{BE} \cong \overline{BD}$



**SOLUTION:**

Proof:

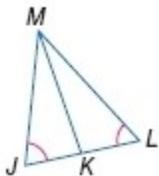


## 4-5 Proving Triangles Congruent - ASA, AAS

12. **PROOF** Write a flow proof.

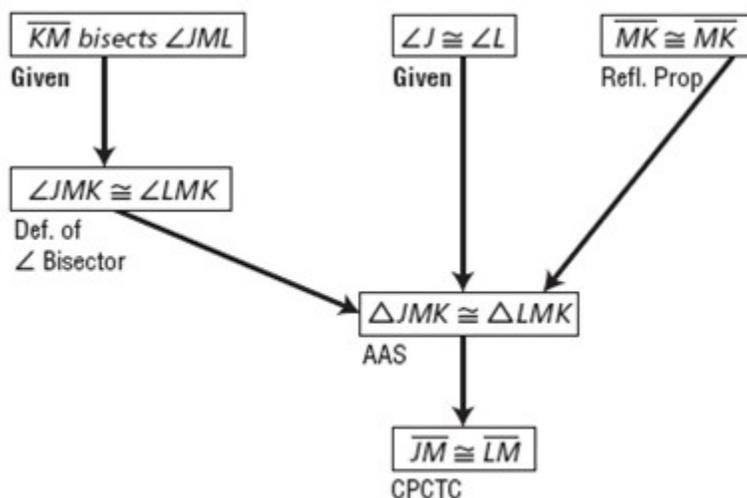
**Given:**  $\overline{KM}$  bisects  $\angle JML$ ;  $\angle J \cong \angle L$ .

**Prove:**  $\overline{JM} \cong \overline{LM}$

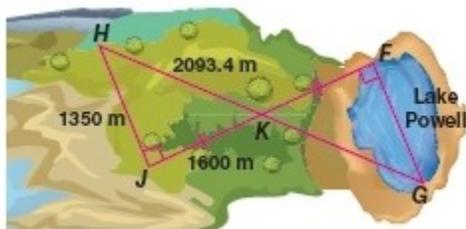


**SOLUTION:**

Proof:



13. **FITNESS** A high school wants to hold a 1500-meter regatta on Lake Powell but is unsure if the lake is long enough. To measure the distance across the lake, the crew members locate the vertices of the triangles below and find the measures of the lengths of  $\triangle HJK$  as shown below.



- Explain how the crew team can use the triangles formed to estimate the distance  $FG$  across the lake.
- Using the measures given, is the lake long enough for the team to use as the location for their regatta? Explain your reasoning.

**SOLUTION:**

a.  $\angle HJK \cong \angle GFK$  since all right angles are congruent.

We are given that  $\overline{JK} \cong \overline{KF}$ .  $\angle HKJ$  and  $\angle FKG$  are vertical angles, so  $\angle HKJ \cong \angle FKG$  by the Vertical Angles Theorem.

By ASA,  $\triangle HJK \cong \triangle GFK$ , so  $\overline{FG} \cong \overline{HJ}$  by CPCTC.

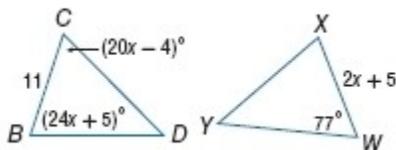
b. Since  $\overline{FG} \cong \overline{HJ}$  and  $HJ = 1350$ ,  $FG = 1350$ .

If the regatta is to be 1500 m, the lake is not long enough, since  $1350 < 1500$ .

#### 4-5 Proving Triangles Congruent - ASA, AAS

**ALGEBRA** Find the value of the variable that yields congruent triangles.

14.  $\triangle BCD \cong \triangle WXY$



**SOLUTION:**

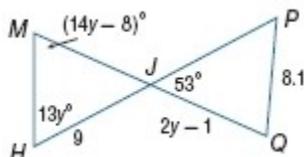
Since  $\triangle BCD \cong \triangle WXY$ , the corresponding sides are congruent. Therefore,  $\overline{BC} \cong \overline{WX}$ . By the definition of congruence,  $BC = WX$ .

$$11 = 2x + 5 \quad BC = WX$$

$$2x = 6 \quad \text{Simplify.}$$

$$x = 3 \quad \text{Divide each side by 2.}$$

15.  $\triangle MHJ \cong \triangle PQJ$



**SOLUTION:**

Since  $\triangle MHJ \cong \triangle PQJ$ , the corresponding sides are congruent. Therefore,  $\overline{HJ} \cong \overline{QJ}$ . By the definition of congruence,  $HJ = QJ$ .

$$9 = 2y - 1 \quad HJ = QJ$$

$$2y = 10 \quad \text{Simplify.}$$

$$y = 5 \quad \text{Divide each side by 2.}$$

16. **THEATER DESIGN** The trusses of the roof of the outdoor theater shown below appear to be several different pairs of congruent triangles. Assume that trusses that appear to lie on the same line actually lie on the same line. **Refer to the figure on page 278.**

a. If  $\overline{AB}$  bisects  $\angle CBD$  and  $\angle CAD$ , prove that  $\triangle ABC \cong \triangle ABD$ .

b. If  $\triangle ABC \cong \triangle ABD$  and  $\angle FCA \cong \angle EDA$ , prove that  $\triangle CAF \cong \triangle DAE$ .

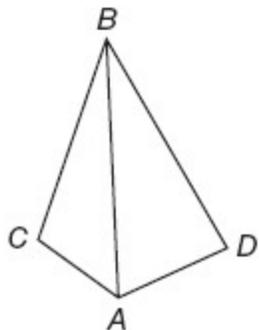
c. If  $\overline{HB} \cong \overline{EB}$ ,  $\angle BHG \cong \angle BEA$ ,  $\angle HGJ \cong \angle EAD$ , and  $\angle JGB \cong \angle DAB$ , prove that  $\triangle BHG \cong \triangle BEA$ .

**SOLUTION:**

a. Given:  $\overline{AB}$  bisects  $\angle CBD$  and  $\angle CAD$ .

Prove:  $\triangle ABC \cong \triangle ABD$ .

#### 4-5 Proving Triangles Congruent - ASA, AAS



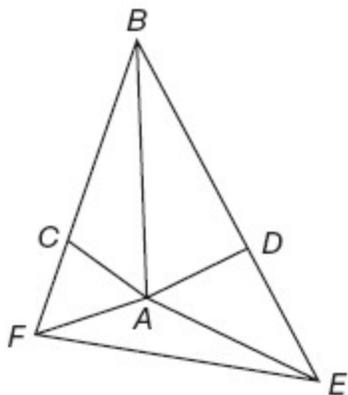
Proof:

Statements (Reasons)

1.  $\overline{AB}$  bisects  $\angle CBD$  and  $\angle CAD$ . (Given)
2.  $\angle ABC \cong \angle ABD$ ,  $\angle CAB \cong \angle DAB$  (Definition of bisect)
3.  $\overline{AB} \cong \overline{AB}$  (Reflexive Property)
4.  $\triangle ABC \cong \triangle ABD$  (ASA)

b. Given:  $\triangle ABC \cong \triangle ABD$ ,  $\angle FCA \cong \angle EDA$

Prove:  $\triangle CAF \cong \triangle DAE$ .



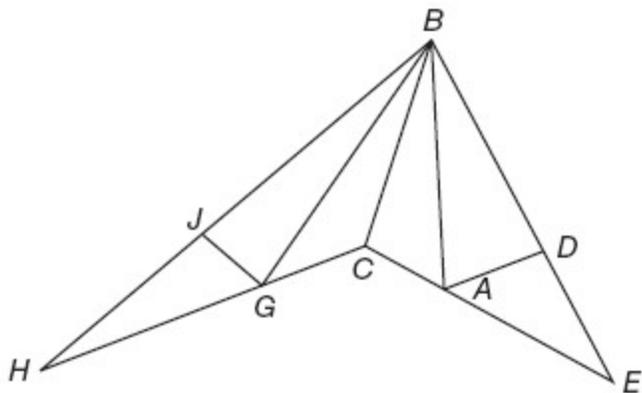
Proof:

Statements (Reasons)

1.  $\triangle ABC \cong \triangle ABD$ ,  $\angle FCA \cong \angle EDA$  (Given)
2.  $\overline{CA} \cong \overline{DA}$  (CPCTC)
3.  $\angle CAF \cong \angle DAE$  (Vertically opposite angles are congruent.)
4.  $\triangle CAF \cong \triangle DAE$  (ASA)

c. Given:  $\overline{HB} \cong \overline{EB}$ ,  $\angle BHG \cong \angle BEA$ ,  $\angle HGJ \cong \angle EAD$ ,  $\angle JGB \cong \angle DAB$

Prove:  $\triangle BHG \cong \triangle BEA$



#### 4-5 Proving Triangles Congruent - ASA, AAS

Proof:

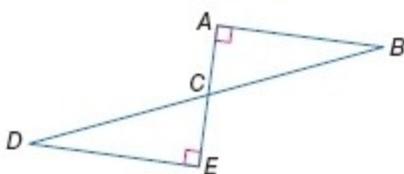
Statements (Reasons)

1.  $\overline{HB} \cong \overline{EB}$ ,  $\angle BHG \cong \angle BEA$ ,  $\angle HGJ \cong \angle EAD$ ,  $\angle JGB \cong \angle DAB$  (Given)
2.  $m\angle HGJ = m\angle EAD$ ,  $m\angle JGB = m\angle DAB$  (Definition of Congruence)
3.  $m\angle HGJ + m\angle JGB = m\angle HGB$ ,  $m\angle EAD + m\angle DAB = m\angle EAB$  (Addition Property of Equality)
4.  $m\angle EAD + m\angle DAB = m\angle HGB$ ,  $m\angle EAD + m\angle DAB = m\angle EAB$  (Angle Addition Postulate)
5.  $m\angle HGB = m\angle EAB$  (Substitution)
6.  $\angle HGB \cong \angle EAB$  (Definition of congruence)
7.  $\triangle BHG \cong \triangle BEA$  (AAS)

**PROOF Write a paragraph proof.**

17. **Given:**  $\overline{AE} \perp \overline{DE}$ ,  $\overline{EA} \perp \overline{AB}$ ,  $C$  is the midpoint of  $\overline{AE}$ .

**Prove:**  $\overline{CD} \cong \overline{CB}$

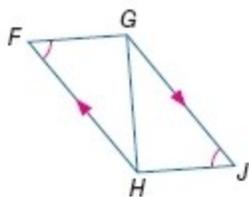


**SOLUTION:**

Proof: We are given that  $\overline{AE}$  is perpendicular to  $\overline{DE}$ ,  $\overline{EA}$  is perpendicular to  $\overline{AB}$ , and  $C$  is the midpoint of  $\overline{AE}$ . Since  $\overline{AE}$  is perpendicular to  $\overline{DE}$ ,  $m\angle CED = 90$ . Since  $\overline{EA}$  is perpendicular to  $\overline{AB}$ ,  $m\angle BAC = 90$ .  $\angle CED \cong \angle BAC$  because all right angles are congruent.  $\overline{AC} \cong \overline{CE}$  from the Midpoint Theorem.  $\angle ECD \cong \angle ACB$  because they are vertical angles. Angle-Side-Angle postulate gives us that  $\triangle CED \cong \triangle CAB$ .  $\overline{CD} \cong \overline{CB}$  because corresponding parts of congruent triangles are congruent.

18. **Given:**  $\angle F \cong \angle J$ ,  $\overline{FH} \parallel \overline{GJ}$

**Prove:**  $\overline{FH} \cong \overline{JG}$



**SOLUTION:**

Proof:  $\angle F \cong \angle J$  and  $\overline{FH} \parallel \overline{GJ}$  because it is given.

$\angle FHG \cong \angle JGH$  because they are alternate interior angles.

By the Reflexive Property,  $\overline{GH} \cong \overline{GH}$ .

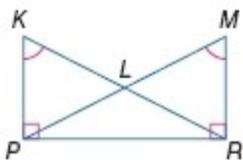
So  $\triangle GHJ \cong \triangle HGF$  by the Angle-Angle-Side postulate. Then  $\overline{FH} \cong \overline{JG}$  since corresponding parts of congruent triangles are congruent.

## 4-5 Proving Triangles Congruent - ASA, AAS

**PROOF** Write a two-column proof.

19. **Given:**  $\angle K \cong \angle M$ ,  $\overline{KP} \perp \overline{PR}$ ,  $\overline{MR} \perp \overline{PR}$

**Prove:**  $\angle KPL \cong \angle MRL$



**SOLUTION:**

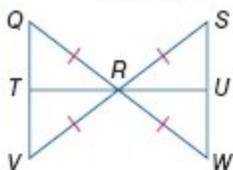
Proof:

Statements (Reasons)

1.  $\angle K \cong \angle M$ ,  $\overline{KP} \perp \overline{PR}$ ,  $\overline{MR} \perp \overline{PR}$  (Given)
2.  $\angle KPR$  and  $\angle MRP$  are both right angles. (Definition of Perpendicular lines.)
3.  $\angle KPR \cong \angle MRP$  (All right angles are congruent.)
4.  $\overline{PR} \cong \overline{PR}$  (Reflexive Prop.)
5.  $\triangle KPR \cong \triangle MRP$  (AAS)
6.  $\overline{KP} \cong \overline{MR}$  (CPCTC)
7.  $\angle KLP \cong \angle MLR$  (Vertical angles are congruent.)
8.  $\triangle KLP \cong \triangle MLR$  (AAS)
9.  $\angle KPL \cong \angle MRL$  (CPCTC)

20. **Given:**  $\overline{QR} \cong \overline{SR} \cong \overline{WR} \cong \overline{VR}$

**Prove:**  $\overline{QT} \cong \overline{WU}$



**SOLUTION:**

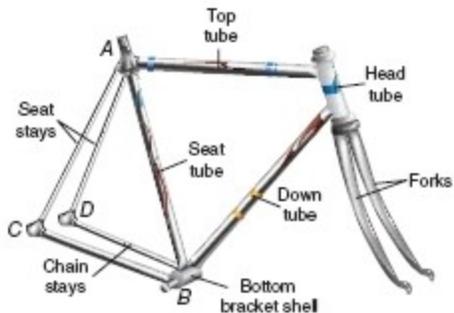
Proof:

Statements (Reasons)

1.  $\overline{QR} \cong \overline{SR} \cong \overline{WR} \cong \overline{VR}$  (Given)
2.  $\angle QRV \cong \angle SRW$  (Vertical angles are congruent.)
3.  $\triangle VRQ \cong \triangle SRW$  (SAS)
4.  $\angle VQR \cong \angle SWR$  (CPCTC)
5.  $\angle QRT \cong \angle URW$  (Vertical angles are congruent.)
6.  $\triangle URW \cong \triangle TRQ$  (ASA)
7.  $\overline{QT} \cong \overline{WU}$  (CPCTC)

## 4-5 Proving Triangles Congruent - ASA, AAS

21. **FITNESS** The seat tube of a bicycle forms a triangle with each seat and chain stay as shown. If each seat stay makes a  $44^\circ$  angle with its corresponding chain stay and each chain stay makes a  $68^\circ$  angle with the seat tube, show that the two seat stays are the same length.



**SOLUTION:**

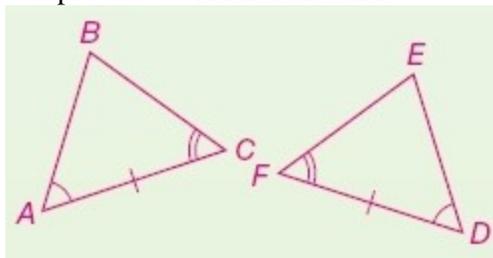
Proof:

Statements (Reasons)

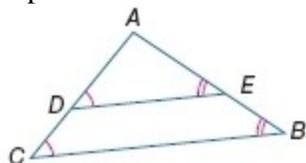
1.  $m\angle ACB = 44$ ,  $m\angle ADB = 44$ ,  $m\angle CBA = 68$ ,  $m\angle DBA = 68$  (Given)
  2.  $m\angle ACB = m\angle ADB$ ,  $m\angle CBA = m\angle DBA$  (Substitution)
  3.  $\angle ACB \cong \angle ADB$ ,  $\angle CBA \cong \angle DBA$  (Definition of congruence)
  4.  $\overline{AB} \cong \overline{AB}$  (Reflexive Property)
  5.  $\triangle ADB \cong \triangle ACB$  (AAS)
  6.  $\overline{AC} \cong \overline{AD}$  (CPCTC)
22. **OPEN ENDED** Draw and label two triangles that could be proved congruent by ASA.

**SOLUTION:**

Sample answer:  $\triangle ABC \cong \triangle DEF$



23. **ERROR ANALYSIS** Tyrone says it is not possible to show that  $\triangle ADE \cong \triangle ACB$ . Lorenzo disagrees, explaining that since  $\angle ADE \cong \angle ACB$ , and  $\angle A \cong \angle A$  by the Reflexive Property,  $\triangle ADE \cong \triangle ACB$ . Is either of them correct? Explain.



**SOLUTION:**

Tyrone is correct.

Lorenzo showed that all three corresponding angles were congruent, but AAA is not a proof of triangle congruence.

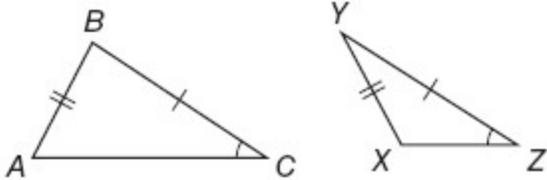
#### 4-5 Proving Triangles Congruent - ASA, AAS

24. **REASONING** Find a counterexample to show why SSA (Side-Side-Angle) cannot be used to prove the congruence of two triangles.

**SOLUTION:**

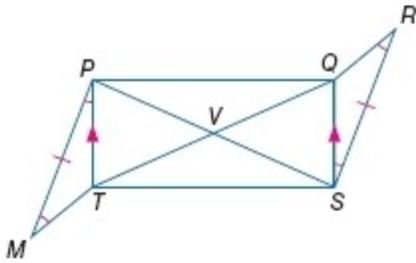
Sample answer: To find a counterexample, show a set of triangles with corresponding SSA congruence and then show that at least one pair of the other 2 corresponding angles are not congruent. If SSA was a valid congruence theorem, then each pair of corresponding angles would be congruent.

Consider triangles  $ABC$  and  $XYZ$ .  $\overline{AB} \cong \overline{XY}$ ,  $\overline{BC} \cong \overline{YZ}$ ,  $\angle C \cong \angle Z$ . However,  $\angle X \not\cong \angle A$ .  $\angle X$  is obtuse while  $\angle A$  is acute so  $\triangle ABC \not\cong \triangle XYZ$ .



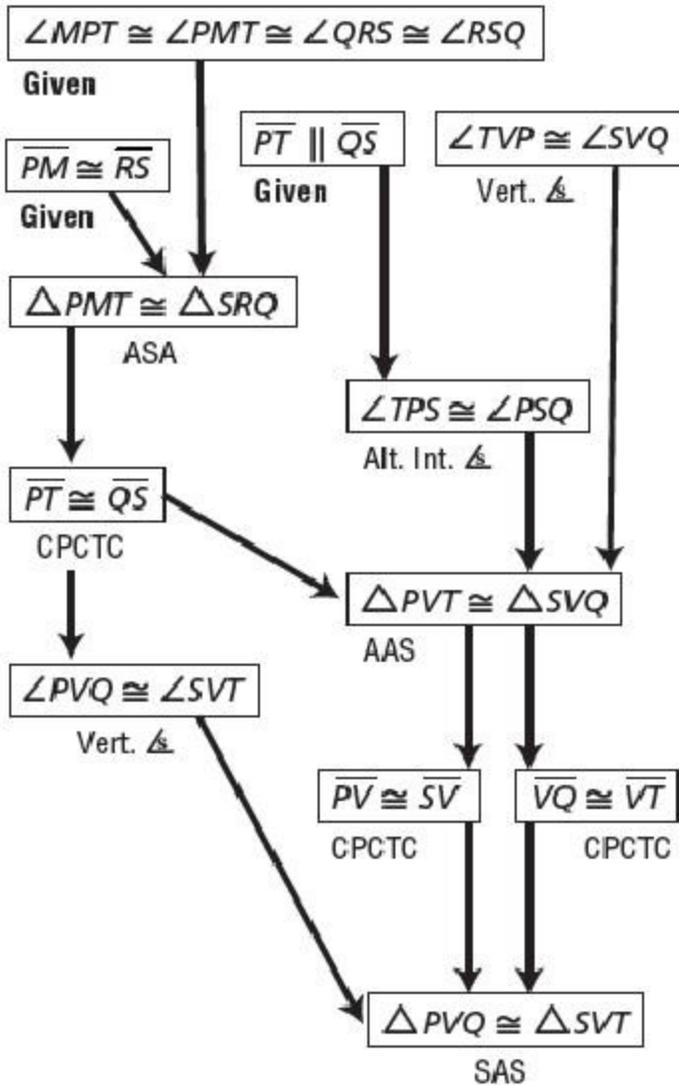
#### 4-5 Proving Triangles Congruent - ASA, AAS

25. **CHALLENGE** Using the information given in the diagram, write a flow proof to show that  $\triangle PVQ \cong \triangle SVT$ .



**SOLUTION:**

Proof:



#### 4-5 Proving Triangles Congruent - ASA, AAS

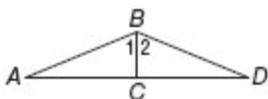
26. How do you know what method (SSS, SAS, etc.) to use when proving triangle congruence? Use a chart to explain your reasoning.

**SOLUTION:**

Sample answer: Identify what sides and/or angles are congruent or could be shown to be congruent. Then select the corresponding method.

Method	Use when...
Definition of Congruent Triangles	All corresponding parts of one triangle are congruent to the corresponding parts of the other triangle.
SSS	The three sides of one triangle must be congruent to the three sides of the other triangle.
SAS	Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.
ASA	Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.
AAS	Two angles and a nonincluded side of one triangle must be congruent to two angles and the corresponding nonincluded side of the other triangle.

27. Given:  $\overline{BC}$  is perpendicular to  $\overline{AD}$ ;  $\angle 1 \cong \angle 2$ .



Which theorem or postulate could be used to prove  $\triangle ABC \cong \triangle DBC$ ?

- A AAS
- B ASA
- C SAS
- D SSS

**SOLUTION:**

Given:  $\angle 1 \cong \angle 2$ ;  $\overline{BC} \perp \overline{AD}$ .

By the definition of perpendicular lines,  $m\angle BCA = m\angle BCD = 90$ . That is,  $\angle BCA \cong \angle BCD$ .

And  $\overline{BC} \cong \overline{BC}$ , by the Reflexive property.

Therefore, by ASA postulate,  $\triangle ABC \cong \triangle DBC$ .

The correct choice is B.

#### 4-5 Proving Triangles Congruent - ASA, AAS

28. **SHORT RESPONSE** Write an expression that can be used to find the values of  $s(n)$  in the table.

$n$	-8	-4	-1	0	1
$s(n)$	1.00	2.00	2.75	3.00	3.25

**SOLUTION:**

Is the expression linear? To check, confirm if there is a constant rate of increase (slope).

As the input value  $n$  increases from -1 to 0, the output value  $s(n)$  increases from 2.75 to 3.00.

As the input value  $n$  increases from 0 to 1, the output value  $s(n)$  increases from 3.00 to 3.25.

We see a constant increase of 0.25 for every increase in input by 1. The same constant can be found when testing other values of  $n$ .

This constant increase of 0.25 is equivalent to a slope  $m$  of  $\frac{1}{4}$ . We now know the expression is linear. The  $y$ -intercept is at (0, 3.00), so the value of  $b$  in  $mx + b$  is 3

Therefore, the expression for  $s(n)$  could be  $\frac{1}{4}n + 3$ .

29. **ALGEBRA** If  $-7$  is multiplied by a number greater than 1, which of the following describes the result?

**F** a number greater than 7

**G** a number between  $-7$  and 7

**H** a number greater than  $-7$

**J** a number less than  $-7$

**SOLUTION:**

When a number greater than 1 is multiplied with  $-7$ , we get a negative number less than  $-7$ .

For example:

$$4 \times -7 = -28$$

$$-28 < -7$$

$$2 \times -7 = -14$$

$$-14 < -7$$

$$1.1 \times -7 = -7.7$$

$$-7.7 < -7$$

The correct choice is J.

#### 4-5 Proving Triangles Congruent - ASA, AAS

30. SAT/ACT  $\sqrt{121+104} = ?$

- A 15
- B 21
- C 25
- D 125
- E 225

**SOLUTION:**

$$\begin{aligned}\sqrt{121+104} &= \sqrt{225} \quad \text{Simplify.} \\ &= 15 \quad \text{Take the square root.}\end{aligned}$$

The correct choice is A.

**Determine whether  $\triangle ABC \cong \triangle XYZ$ . Explain.**

31.  $A(6, 4)$ ,  $B(1, -6)$ ,  $C(-9, 5)$ ,  $X(0, 7)$ ,  $Y(5, -3)$ ,  $Z(15, 8)$

**SOLUTION:**

Use the distance formula to find the length of each side of the triangles.

The side lengths of the triangle  $ABC$  are:

$$AB = \sqrt{(1-6)^2 + (-6-4)^2} = \sqrt{125};$$

$$BC = \sqrt{(-9-1)^2 + (5+6)^2} = \sqrt{221};$$

$$CA = \sqrt{(-9-6)^2 + (5-4)^2} = \sqrt{226}$$

The side lengths of the triangle  $XYZ$  are:

$$XY = \sqrt{(5-0)^2 + (-3-7)^2} = \sqrt{125};$$

$$YZ = \sqrt{(15-5)^2 + (8+3)^2} = \sqrt{221};$$

$$ZX = \sqrt{(0-15)^2 + (7-8)^2} = \sqrt{226}$$

$$\overline{AB} \cong \overline{XY}; \overline{BC} \cong \overline{YZ}; \overline{AC} \cong \overline{ZX}$$

The corresponding sides have the same measure and are congruent.  $\triangle ABC \cong \triangle XYZ$  by SSS.

#### 4-5 Proving Triangles Congruent - ASA, AAS

32.  $A(0, 5)$ ,  $B(0, 0)$ ,  $C(-2, 0)$ ,  $X(4, 8)$ ,  $Y(4, 3)$ ,  $Z(6, 3)$

**SOLUTION:**

Use the distance formula to find the length of each side of the triangles.

The side lengths of the triangle  $ABC$  are:

$$AB = \sqrt{(0-0)^2 + (0-5)^2} = 5;$$

$$BC = \sqrt{(-2-0)^2 + (0-0)^2} = 2;$$

$$CA = \sqrt{(-2-0)^2 + (0-5)^2} = \sqrt{29};$$

The side lengths of the triangle  $XYZ$  are:

$$XY = \sqrt{(4-4)^2 + (3-8)^2} = 5;$$

$$YZ = \sqrt{(6-4)^2 + (3-3)^2} = 2;$$

$$ZX = \sqrt{(6-4)^2 + (3-8)^2} = \sqrt{29};$$

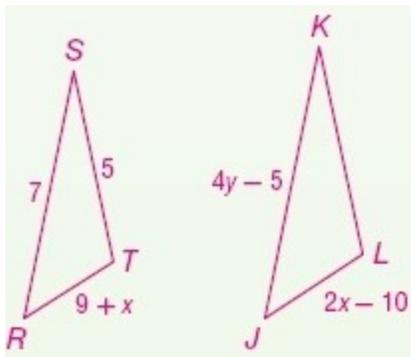
$$\overline{AB} \cong \overline{XY}; \overline{BC} \cong \overline{YZ}; \overline{AC} \cong \overline{ZX};$$

The corresponding sides have the same measure and are congruent.  $\triangle ABC \cong \triangle XYZ$  by SSS.

#### 4-5 Proving Triangles Congruent - ASA, AAS

33. **ALGEBRA** If  $\triangle RST \cong \triangle JKL$ ,  $RS = 7$ ,  $ST = 5$ ,  $RT = 9 + x$ ,  $JL = 2x - 10$ , and  $JK = 4y - 5$ , draw and label a figure to represent the congruent triangles. Then find  $x$  and  $y$ .

**SOLUTION:**



Corresponding sides of triangles  $RST$  and  $JKL$  are congruent.

Since  $\overline{RS} \cong \overline{JK}$ ,  $RS = JK$ .

Solve the equation for  $y$ .

$$7 = 4y - 5 \quad RS = JK$$

$$12 = 4y \quad \text{Add 5 to each side.}$$

$$y = 3 \quad \text{Divide each side by 4.}$$

Since  $\overline{RT} \cong \overline{JL}$ ,  $RT = JL$ .

Solve for  $x$ .

$$9 + x = 2x - 10 \quad RT = JL$$

$$9 + 10 = 2x - x \quad \text{Simplify.}$$

$$x = 19 \quad \text{Simplify.}$$

34. **BUSINESS** Maxine charges \$5 to paint a mailbox and \$4 per hour to mow a lawn. Write an equation to represent the amount of money Maxine can earn from a homeowner who has his or her mailbox painted and lawn mowed.

**SOLUTION:**

Assume that  $x$  represents the time Maxine spends mowing a lawn and let  $y$  be the amount of money Maxine can earn. The expression that represents the amount of money Maxine can earn could be:  $y = 4(x) + 5$ .

## 4-5 Proving Triangles Congruent - ASA, AAS

Copy and complete each truth table.

$p$	$q$	$\sim p$	$\sim p \vee q$
F	T		
T	T		
F	F		
T	F		

35.

SOLUTION:

$p$	$q$	$\sim p$	$\sim p \vee q$
F	T	T	T
T	T	F	T
F	F	T	T
T	F	F	F

$p$	$q$	$\sim q$	$\sim q \wedge p$
F		F	
T		T	
T		F	
F		T	

36.

SOLUTION:

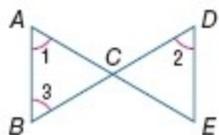
$p$	$q$	$\sim q$	$\sim q \wedge p$
F	T	F	F
T	F	T	T
T	T	F	F
F	F	T	F

**PROOF** Write a two-column proof for each of the following.

37. Given:  $\angle 2 \cong \angle 1$

$$\angle 1 \cong \angle 3$$

Prove:  $\overline{AB} \parallel \overline{DE}$



SOLUTION:

Proof:

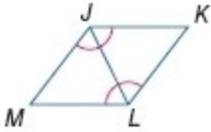
Statements (Reasons)

- $\angle 2 \cong \angle 1, \angle 1 \cong \angle 3$  (Given)
- $\angle 2 \cong \angle 3$  (Transitive Property)
- $\overline{AB} \parallel \overline{DE}$  (If alternative interior angles are congruent, then the lines are parallel.)

## 4-5 Proving Triangles Congruent - ASA, AAS

38. **Given:**  $\angle MJK \cong \angle KLM$   
 $\angle LMJ$  and  $\angle KLM$  are supplementary.

**Prove:**  $\overline{KJ} \parallel \overline{LM}$



**SOLUTION:**

Proof:

Statements (Reasons)

- $\angle MJK \cong \angle KLM$ ,  $\angle LMJ$  and  $\angle KLM$  are supplementary. (Given)
- $m\angle MJK = m\angle KLM$  (Definition of congruent angles.)
- $m\angle LMJ + m\angle KLM = 180$  (Definition of supplementary angles)
- $m\angle LMJ + m\angle MJK = 180$  (Substitution)
- $\angle LMJ$  and  $\angle MJK$  are supplementary. (Definition of supplementary angles)
- $\overline{KJ} \parallel \overline{LM}$  (If consecutive interior angles are supplementary, then the lines are parallel.)