8-1 Geometric Mean

Find the geometric mean between each pair of numbers.

1. 5 and 20

SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.
Therefore, the geometric mean of 5 and 20 is
$\sqrt{(5)(20)} = \sqrt{100} = 10.$

2. 36 and 4

SOLUTION:

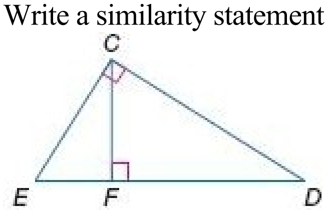
By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.
Therefore, the geometric mean of 36 and 4 is
$\sqrt{(36)(4)} = \sqrt{144} = 12.$

3. 40 and 15

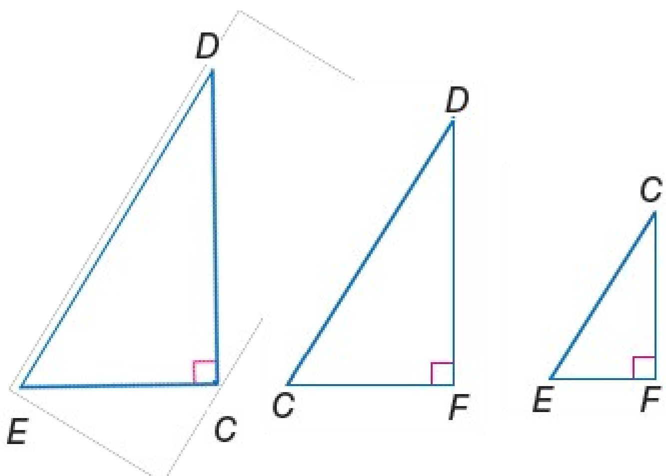
SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.
Therefore, the geometric mean of 40 and 15 is $\sqrt{(40)(15)} = \sqrt{600} = 10\sqrt{6} \approx 24.5.$

4. Write a similarity statement identifying the three similar triangles in the figure.

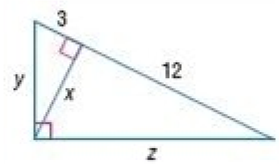


SOLUTION:

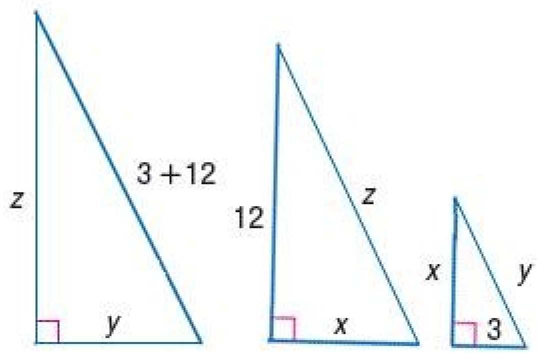


If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other. $\overline{CF}$ is the altitude to the hypotenuse of the right triangle *CED.* Therefore,
$\Delta ECD \sim \Delta CFD \sim \Delta EFC$.

Find x, y, and z.

5. 

SOLUTION:



By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

Solve for *x*.

$$\begin{aligned}\frac{\text{shorter leg}}{\text{longer leg}} &= \frac{\text{shorter leg}}{\text{longer leg}}\\ \frac{3}{x} &= \frac{x}{12}\\ x^2 &= 36\\ x &= 6\end{aligned}$$

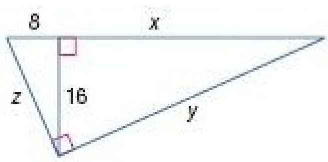
By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Solve for *y*.

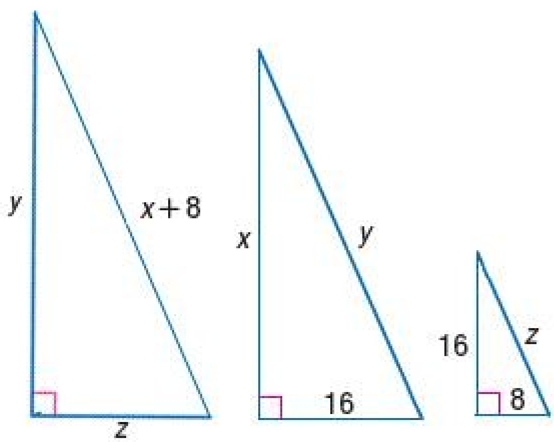
$$\begin{aligned}\frac{\text{hypotenuse}}{\text{shorter leg}} &= \frac{\text{hypotenuse}}{\text{shorter leg}}\\ \frac{15}{y} &= \frac{y}{3}\\ y^2 &= 45\\ y &= \sqrt{45}\\ y &= 3\sqrt{5}\\ y &\approx 6.7\end{aligned}$$

Solve for *z*.

$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{hypotenuse}}{\text{longer leg}}$$
$$\frac{15}{z} = \frac{z}{12}$$
$$z^2 = 180$$
$$z = \sqrt{180}$$
$$z = 6\sqrt{5}$$
$$z \approx 13.4$$

6. 

SOLUTION:



By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

Solve for *x*.

$$\frac{\text{longer leg}}{\text{shorter leg}} = \frac{\text{longer leg}}{\text{shorter leg}}$$
$$\frac{16}{8} = \frac{x}{16}$$
$$16^2 = 8x$$
$$256 = 8x$$
$$\frac{256}{8} = \frac{8x}{8}$$
$$32 = x$$

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the

hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

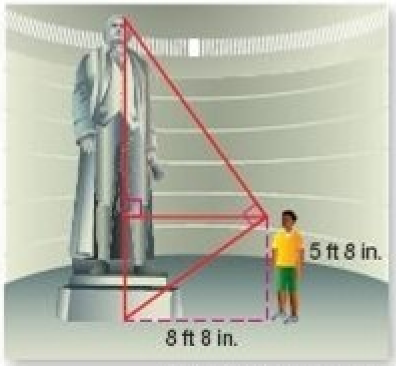
Solve for *y*.

$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{hypotenuse}}{\text{longer leg}}$$
$$\frac{40}{y} = \frac{y}{32}$$
$$y^2 = 1280$$
$$y = \sqrt{1280}$$
$$= 16\sqrt{5}$$
$$\approx 35.8$$

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.
Solve for z.

$$\frac{\text{hypotenuse}}{\text{shorter leg}} = \frac{\text{hypotenuse}}{\text{shorter leg}}$$
$$\frac{40}{z} = \frac{z}{8}$$
$$z^2 = 320$$
$$z = \sqrt{320}$$
$$= 8\sqrt{5}$$
$$\approx 17.9$$

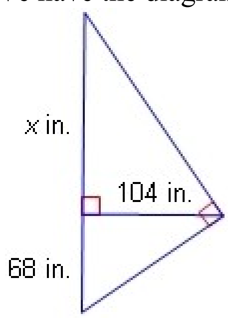
7. **MONUMENTS** Corey is visiting the Jefferson Memorial with his family. He wants to estimate the height of the statue of Thomas Jefferson. Corey stands so that his line of vision to the top and base of the statue form a right angle as shown in the diagram. About how tall is the statue?



Note: Not drawn to scale.

SOLUTION:

We have the diagram as shown.



By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

$$104 = \sqrt{x \cdot 68}$$
$$104^2 = \left(\sqrt{x \cdot 68}\right)^2$$
$$10816 = x \cdot 68$$
$$\frac{10816}{68} = \frac{x \cdot 68}{68}$$
$$159 \approx x$$

So, the total height of the statue is about 159 + 68 or 227 inches, which is equivalent to 18 ft 11 in.

Find the geometric mean between each pair of numbers.

8. 81 and 4

SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.

Therefore, the geometric mean of 81 and 4 is

$\sqrt{(81)(4)} = \sqrt{324} = 18.$

9. 25 and 16

SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$. Therefore, the geometric mean of 25 and 16 is
$\sqrt{(25)(16)} = \sqrt{400} = 20.$

10. 20 and 25

SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$. Therefore, the geometric mean of 20 and 25 is
$\sqrt{(20)(25)} = \sqrt{500} = 10\sqrt{5} \approx 22.4.$

11. 36 and 24

SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$. Therefore, the geometric mean of 36 and 24 is
$\sqrt{(36)(24)} = \sqrt{864} = 12\sqrt{6} \approx 29.4.$

12. 12 and 2.4

SOLUTION:

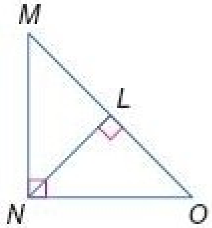
By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$. Therefore, the geometric mean of 12 and 2.4 is
$\sqrt{(12)(2.4)} = \sqrt{28.8} = \frac{12\sqrt{5}}{5} \approx 5.4.$

13. 18 and 1.5

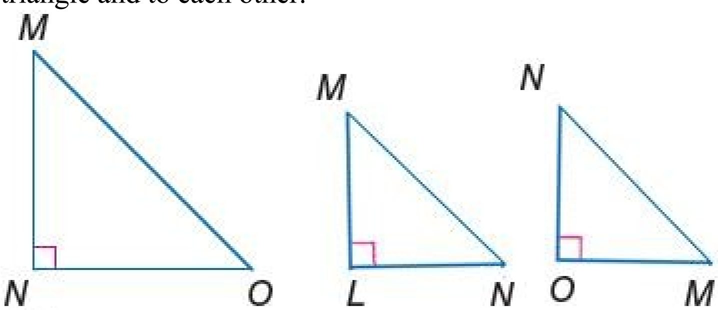
SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$. Therefore, the geometric mean of 18 and 1.5 is
$\sqrt{(18)(1.5)} = \sqrt{27} = 3\sqrt{3} \approx 5.2.$

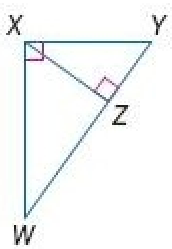
Write a similarity statement identifying the three similar triangles in the figure.

14. 

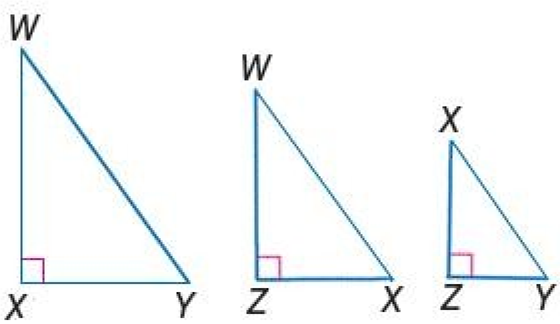
SOLUTION:
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



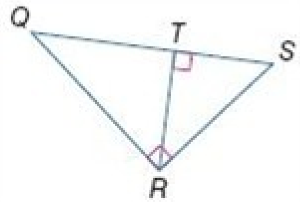
$\overline{LN}$ is the altitude to the hypotenuse of the right triangle *MNO*. Therefore, $\Delta MNO \sim \Delta NLO \sim \Delta MLN$.

15. 

SOLUTION:
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

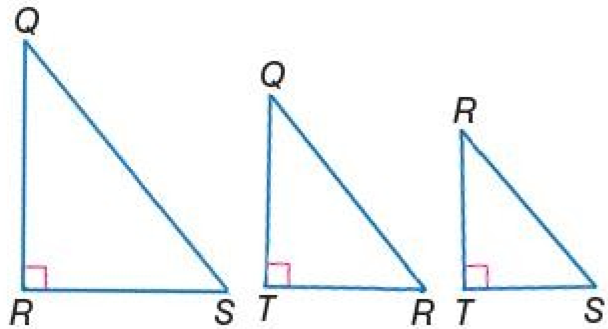


$\overline{XZ}$ is the altitude to the hypotenuse of the right triangle *XYW*. Therefore, $\Delta WXY \sim \Delta XZY \sim \Delta WZX$.

16. 

SOLUTION:

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

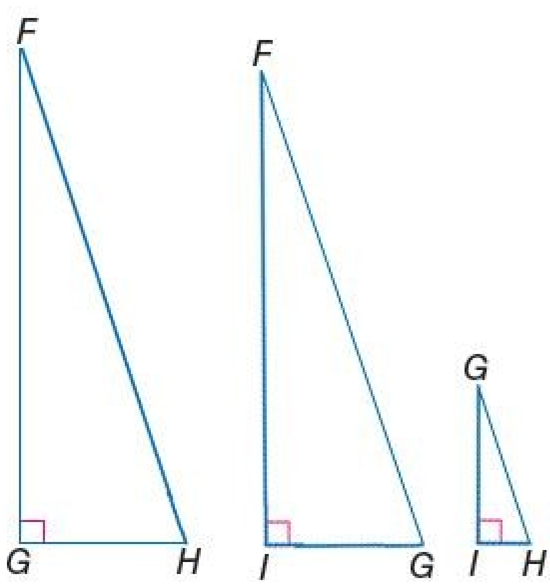


$\overline{RT}$ is the altitude to the hypotenuse of the right triangle *QRS*. Therefore, $\Delta QRS \sim \Delta RTS \sim \Delta QTR$.

17. 

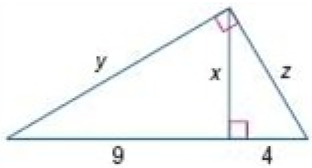
SOLUTION:

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

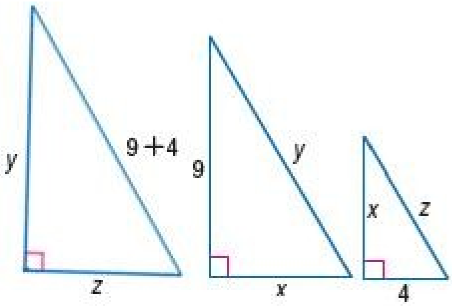


$\overline{GI}$ is the altitude to the hypotenuse of the right triangle *HGF*. Therefore, $\Delta HGF \sim \Delta HIG \sim \Delta GIF$.

8-1 Geometric Mean

18. 

SOLUTION:



By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

Solve for *x*.

$$\frac{\text{longer leg}}{\text{shorter leg}} = \frac{\text{longer leg}}{\text{shorter leg}}$$
$$\frac{x}{4} = \frac{9}{x}$$
$$x^2 = 9 \cdot 4$$
$$x = \sqrt{36}$$
$$x = 6$$

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

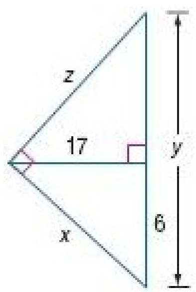
Solve for *y*.

$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{hypotenuse}}{\text{longer leg}}$$
$$\frac{13}{y} = \frac{y}{9}$$
$$y^2 = 117$$
$$y = \sqrt{117}$$
$$= 3\sqrt{13}$$
$$\approx 10.8$$

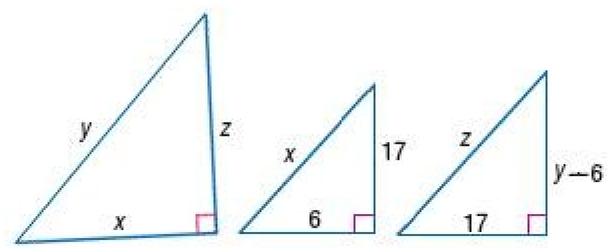
By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Solve for *z*.

$$\frac{\text{hypotenuse}}{\text{shorter leg}} = \frac{\text{hypotenuse}}{\text{shorter leg}}$$
$$\frac{13}{z} = \frac{z}{4}$$
$$z^2 = 52$$
$$z = \sqrt{52}$$
$$= 2\sqrt{13}$$
$$\approx 7.2$$

19. 

SOLUTION:



By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

Solve for *y*.

$$\frac{\text{shorter leg}}{\text{longer leg}} = \frac{\text{shorter leg}}{\text{longer leg}}$$
$$\frac{17}{y-6} = \frac{6}{17}$$
$$17^2 = 6 \cdot (y-6)$$
$$17 = \sqrt{6 \cdot (y-6)}$$
$$289 = 6(y-6)$$
$$48.2 \approx y - 6$$
$$54.2 \approx y$$

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the

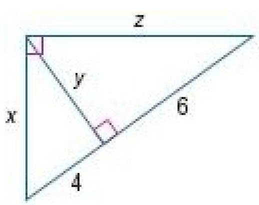
hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg. So,

Solve for *x*.

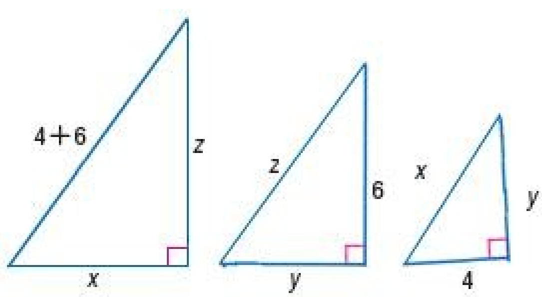
$$\frac{\text{hypotenuse}}{\text{shorter leg}} = \frac{\text{hypotenuse}}{\text{shorter leg}}$$
$$\frac{54.2}{x} = \frac{x}{6}$$
$$x^2 \approx 54.2 \cdot 6$$
$$x^2 \approx 325.2$$
$$x \approx \sqrt{325.2}$$
$$\approx 18.0$$

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.
Solve for *z*.

$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{hypotenuse}}{\text{longer leg}}$$
$$\frac{54.2}{z} \approx \frac{z}{48.2}$$
$$z^2 \approx 54.2 \cdot 48.2$$
$$z^2 \approx 2612.44$$
$$z \approx \sqrt{2612.44}$$
$$\approx 51.1$$

20. 

SOLUTION:



By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

Solve for y.

$$\frac{\text{longer leg}}{\text{shorter leg}} = \frac{\text{longer leg}}{\text{shorter leg}}$$

$$\frac{y}{4} = \frac{6}{y}$$

$$y^2 = 4 \cdot 6$$

$$y^2 = 24$$

$$y = \sqrt{24}$$

$$\approx 4.9$$

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Solve for x.

$$\frac{\text{short leg}}{\text{long leg}} = \frac{\text{short leg}}{\text{long leg}}$$

$$\frac{3}{x} = \frac{x}{12}$$

$$\frac{10}{x} = \frac{x}{4}$$

$$x^2 = 40$$

$$x = \sqrt{40}$$

$$= 4\sqrt{10}$$

$$\approx 6.3$$

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Solve for z.

$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{hypotenuse}}{\text{longer leg}}$$

$$\frac{10}{z} = \frac{z}{6}$$

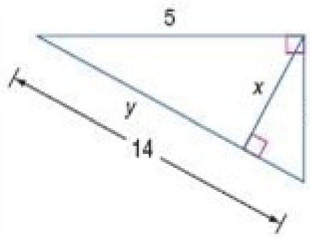
$$z^2 = 6 \cdot 10$$

$$z^2 = 60$$

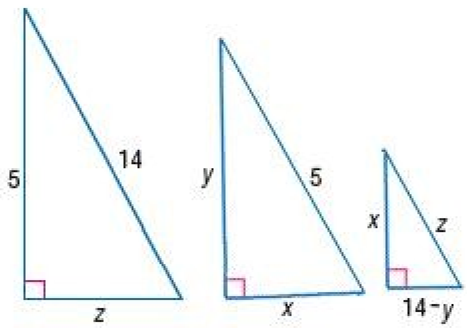
$$z = \sqrt{60}$$

$$= 2\sqrt{15}$$

$$\approx 7.7$$

21. 

SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Solve for *y*.

$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{hypotenuse}}{\text{longer leg}}$$

$$\frac{14}{5} = \frac{5}{y}$$

$$14y = 25$$

$$y = \frac{25}{14}$$

$$y \approx 1.8$$

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

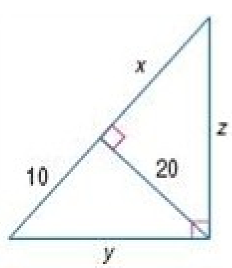
Use the value of *y* to solve the second proportion.

$$\frac{14}{z} = \frac{z}{14 - y}$$

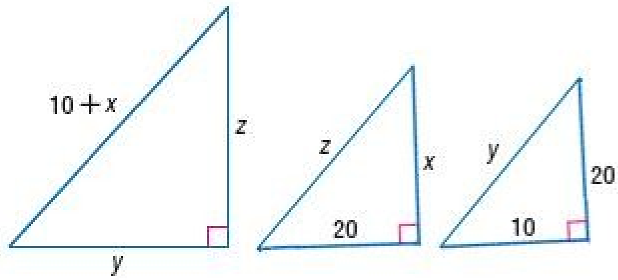
$$\frac{\text{hypotenuse}}{\text{shorter leg}} = \frac{\text{hypotenuse}}{\text{shorter leg}}$$
$$\frac{14}{z} = \frac{z}{14 - y}$$
$$14(14 - y) = z^2$$
$$14(14 - 1.8) \approx z^2$$
$$14 \cdot 12.2 \approx z^2$$
$$170.8 \approx z^2$$
$$\sqrt{170.8} \approx z$$
$$13.1 \approx z$$

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments. So,
Solve for *x*.

$$\frac{\text{longer leg}}{\text{shorter leg}} = \frac{\text{longer leg}}{\text{shorter leg}}$$
$$\frac{x}{14 - y} = \frac{y}{x}$$
$$\frac{x}{12.2} \approx \frac{1.8}{x}$$
$$x^2 \approx 1.8 \cdot 12.2$$
$$x \approx \sqrt{(1.8)(12.2)}$$
$$\approx \sqrt{21.96}$$
$$\approx 4.7$$

22. 

SOLUTION:



By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

Solve for *x*.

$$\frac{\text{shorter leg}}{\text{longer leg}} = \frac{\text{shorter leg}}{\text{longer leg}}$$
$$\frac{20}{x} = \frac{10}{20}$$
$$20^2 = 10 \cdot x$$
$$400 = 10 \cdot x$$
$$\frac{400}{10} = \frac{10x}{10}$$
$$40 = x$$

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

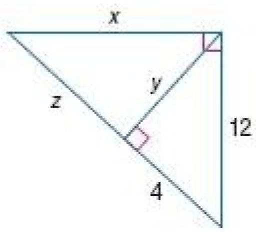
Solve for *y*.

$$\frac{\text{hypotenuse}}{\text{shorter leg}} = \frac{\text{hypotenuse}}{\text{shorter leg}}$$
$$\frac{10 + x}{y} = \frac{y}{10}$$
$$\frac{50}{y} = \frac{y}{10}$$
$$y^2 = 500$$
$$y = \sqrt{500}$$
$$= 10\sqrt{5}$$
$$\approx 22.4$$

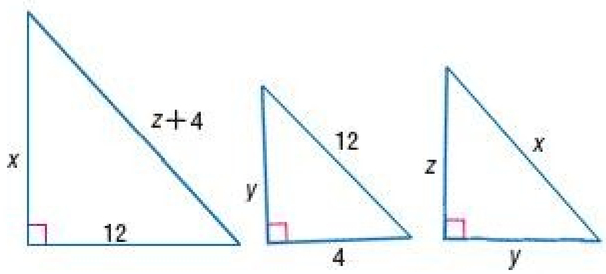
By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Solve for *z*.

$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{hypotenuse}}{\text{longer leg}}$$
$$\frac{10+x}{z} = \frac{z}{40}$$
$$\frac{50}{z} = \frac{z}{40}$$
$$z^2 = 2000$$
$$z = \sqrt{2000}$$
$$= 20\sqrt{5}$$
$$\approx 44.7$$

23. 

SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Solve for z.

$$\frac{\text{shorter leg}}{\text{hypotenuse}} = \frac{\text{shorter leg}}{\text{hypotenuse}}$$
$$\frac{4}{12} = \frac{12}{z+4}$$
$$4(z+4) = 12 \cdot 12$$
$$4z + 16 = 144$$
$$4z + 16 - 16 = 144 - 14$$
$$4z = 128$$
$$z = 32$$

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the

hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Use the value of z to solve the second proportion for x.

$$\frac{\text{longer leg}}{\text{hypotenuse}} = \frac{\text{longer leg}}{\text{hypotenuse}}$$

$$\frac{z}{x} = \frac{x}{z+4}$$

$$\frac{32}{x} = \frac{x}{32+4}$$

$$x^2 = 32(32+4)$$

$$x^2 = 1152$$

$$x = 24\sqrt{2}$$

$$\approx 33.9$$

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

Solve for y.

$$\frac{\text{shorter leg}}{\text{longer leg}} = \frac{\text{shorter leg}}{\text{longer leg}}$$

$$\frac{4}{y} = \frac{y}{z}$$

$$y^2 = z \cdot 4$$

$$y^2 = 32 \cdot 4$$

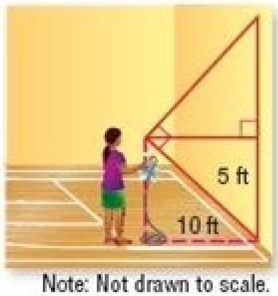
$$y^2 = 128$$

$$y = \sqrt{128}$$

$$y = 8\sqrt{2}$$

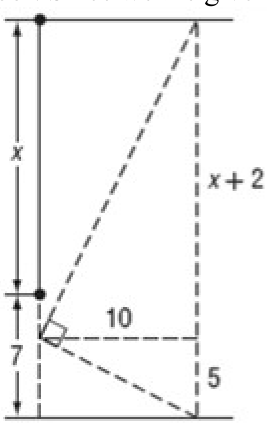
$$y \approx 11.3$$

24. **EVENT PLANNING** Evelina is hanging silver stars from the gym ceiling using string for the homecoming dance. She wants the ends of the strings where the stars will be attached to be 7 feet from the floor. Use the diagram to determine how long she should make the strings.



SOLUTION:

Let x represent the length of the string. Since the star will be 7 feet from the floor, $x + 7$ is the total length of string to floor. Since we are given 5 feet from the floor in the diagram. The distance to the 5 ft point will be $x + 2$.

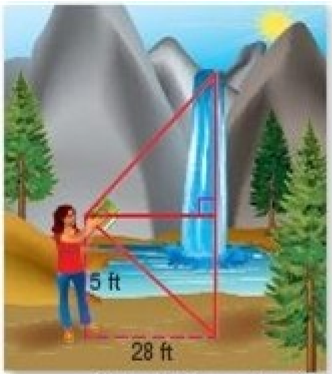


Use the Geometric Mean (Altitude) Theorem to find x.

$$10 = \sqrt{5(x+2)}$$
$$10^2 = \left(\sqrt{5(x+2)}\right)^2$$
$$100 = 5(x+2)$$
$$100 = 5x + 10$$
$$100 - 10 = 5x$$
$$90 = 5x$$
$$\frac{90}{5} = \frac{5x}{5}$$
$$x = 18$$

So she should make the strings of length 18 feet.

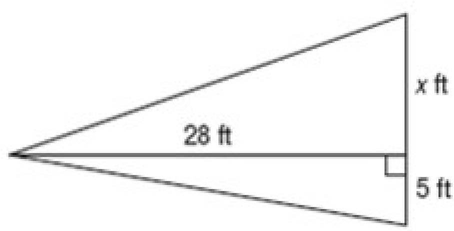
25. **WATERFALLS** Makayla is using a book to sight the top of a waterfall. Her eye level is 5 feet from the ground and she is a horizontal distance of 28 feet from the waterfall. Find the height of the waterfall to the nearest tenth of a foot.



Note: Not drawn to scale.

SOLUTION:

We have the diagram as shown.



By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

$$28 = \sqrt{x \cdot 5}$$
$$28^2 = \left(\sqrt{x \cdot 5}\right)^2$$
$$784 = x \cdot 5$$
$$\frac{784}{5} = \frac{5x}{5}$$
$$156.8 = x$$

So, the total height of the waterfall is 156.8 + 5 = 161.8 ft.

Find the geometric mean between each pair of numbers.

26. $\frac{1}{5}$ and 60

SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.

Therefore, the geometric mean of $\frac{1}{5}$ and 60 is

$$\sqrt{\left(\frac{1}{5}\right)(60)} = \sqrt{12} = 2\sqrt{3} \approx 3.5.$$

27. $\frac{3\sqrt{2}}{7}$ and $\frac{5\sqrt{2}}{7}$

SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.

Therefore, the geometric mean of $\frac{3\sqrt{2}}{7}$ and $\frac{5\sqrt{2}}{7}$ is

$$\sqrt{\left(\frac{3\sqrt{2}}{7}\right)\left(\frac{5\sqrt{2}}{7}\right)} = \sqrt{\frac{15 \cdot 2}{49}} = \frac{\sqrt{30}}{7} \approx 0.8.$$

28. $\frac{3\sqrt{5}}{4}$ and $\frac{5\sqrt{5}}{4}$

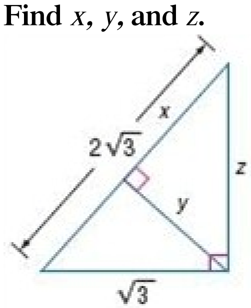
SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.

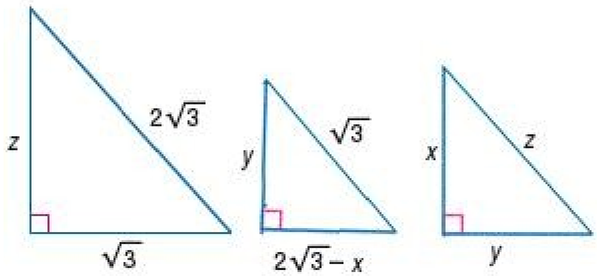
Therefore, the geometric mean of $\frac{3\sqrt{5}}{4}$ and $\frac{5\sqrt{5}}{4}$ is

$$\sqrt{\left(\frac{3\sqrt{5}}{4}\right)\left(\frac{5\sqrt{5}}{4}\right)} = \sqrt{\frac{15 \cdot 5}{16}} = \frac{5\sqrt{3}}{4} \approx 2.2.$$

Find *x*, *y*, and *z*.

29. 

SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Solve for *x*.

$$\frac{\text{hypotenuse}}{\text{shorter leg}} = \frac{\text{hypotenuse}}{\text{shorter leg}}$$

$$\frac{2\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2\sqrt{3} - x}$$

$$2\sqrt{3}\left(2\sqrt{3} - x\right) = \sqrt{3} \cdot \sqrt{3}$$

$$12 - 2\sqrt{3}x = 3$$

$$12 - 12 - 2\sqrt{3}x = 3 - 12$$

$$-2\sqrt{3}x = -9$$

$$\frac{-2\sqrt{3}x}{-2\sqrt{3}} = \frac{-9}{-2\sqrt{3}}$$

$$x = \frac{9}{2\sqrt{3}} = \frac{3\sqrt{3}}{2} \approx 2.6$$

Use the value of *x* to solve the for *z*.

$$\frac{2\sqrt{3}}{z} = \frac{z}{x}.$$

$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{hypotenuse}}{\text{longer leg}}$$

$$\frac{2\sqrt{3}}{z} = \frac{z}{x}$$

$$z^2 = 2\sqrt{3}\left(\frac{3\sqrt{3}}{2}\right)$$

$$z^2 = \frac{(2 \cdot 3)\sqrt{3}\sqrt{3}}{2}$$

$$z^2 = 9$$

$$z = 3$$

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

Solve for y.

$$\frac{\text{shorter leg}}{\text{longer leg}} = \frac{\text{shorter leg}}{\text{longer leg}}$$

$$\frac{2\sqrt{3} - x}{y} = \frac{y}{x}$$

$$y^2 = x \cdot \left(2\sqrt{3} - x\right)$$

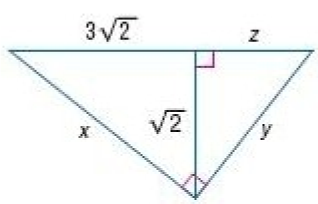
$$y^2 = \frac{3\sqrt{3}}{2} \cdot \left(2\sqrt{3} - \frac{3\sqrt{3}}{2}\right)$$

$$y^2 = \left(\frac{3\sqrt{3}}{2}\right)\left(\frac{4\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}\right)$$

$$y^2 = \left(\frac{3\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

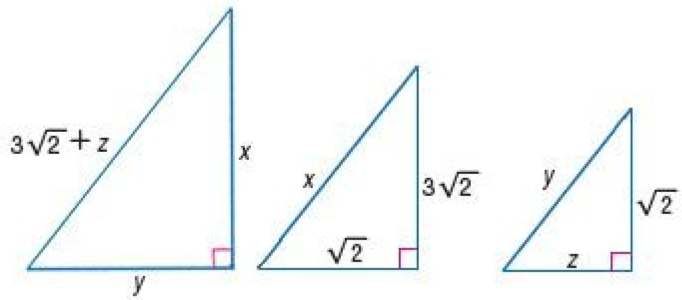
$$y^2 = \frac{9}{4}$$

$$y = \frac{3}{2}$$

30. 

SOLUTION:

8-1 Geometric Mean



By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

Solve for *z*.

$$\frac{\text{longer leg}}{\text{shorter leg}} = \frac{\text{longer leg}}{\text{shorterer leg}}$$

$$\frac{3\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{z}$$

$$\left(\sqrt{2}\right)^2 = 3\sqrt{2} \cdot z$$

$$2 = 3\sqrt{2} \cdot z$$

$$\frac{2}{3\sqrt{2}} = z$$

$$\frac{\sqrt{2}}{3} = z$$

$$z \approx 0.5$$

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Solve for *x*.

$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{shorter leg}}{\text{longer leg}}$$

$$\frac{3\sqrt{2}+z}{x} = \frac{x}{3\sqrt{2}}$$

$$\frac{\left(3\sqrt{2}+\frac{\sqrt{2}}{3}\right)}{x} = \frac{x}{3\sqrt{2}}$$

$$\frac{\left(\frac{10\sqrt{2}}{3}\right)}{x} = \frac{x}{3\sqrt{2}}$$

$$x^2 = \frac{10\sqrt{2}}{3} \cdot 3\sqrt{2}$$

$$x^2 = 20$$

$$x = \sqrt{20}$$

$$x = 2\sqrt{5}$$

$$\approx 4.5$$

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Solve for y.

$$\frac{\text{hypotenuse}}{\text{shorter leg}} = \frac{\text{hypotenuse}}{\text{shorter leg}}$$

$$\frac{\left(3\sqrt{2}+z\right)}{y} = \frac{y}{z}$$

$$\frac{\left(3\sqrt{2}+\frac{\sqrt{2}}{3}\right)}{y} = \frac{y}{\left(\frac{\sqrt{2}}{3}\right)}$$

$$\frac{\left(\frac{10\sqrt{2}}{3}\right)}{y} = \frac{y}{\left(\frac{\sqrt{2}}{3}\right)}$$

$$y^2 = \frac{10\sqrt{2}}{3} \cdot \frac{\sqrt{2}}{3}$$

$$y^2 = \frac{20}{9}$$

$$y = \sqrt{\frac{20}{9}}$$

$$y = \frac{2\sqrt{5}}{3}$$

$$\approx 1.5$$

31. **ALGEBRA** The geometric mean of a number and four times the number is 22. What is the number?

SOLUTION:

Let x be the first number. Then the other number will be $4x$. By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$. So,

$22 = \sqrt{x \cdot 4x} = \sqrt{4x^2}$.

$22 = 2x$

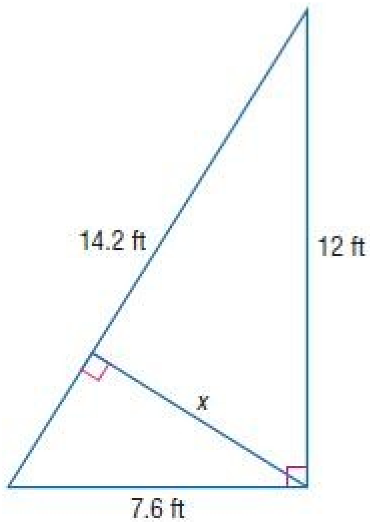
$11 = x$

Therefore, the number is 11.

Use similar triangles to find the value of x.

32. Refer to the figure on page 537.

SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg. Let *y* be the shorter segment of the hypotenuse of the bigger right triangle.

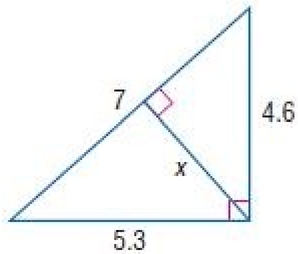
$$\frac{\text{Hypotenuse}}{\text{Shorter Leg}} = \frac{\text{Hypotenuse}}{\text{Shorter Leg}}$$
$$\frac{14.2}{7.6} = \frac{7.6}{y}$$
$$14.2y = 57.76$$
$$y \approx 4.1$$

So, the shorter segment of the right triangle is about 14.2-4.1=10.1 ft.

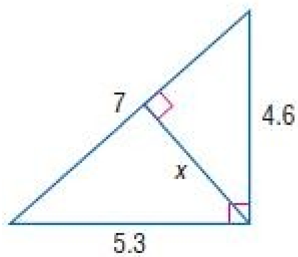
By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

$$\frac{\text{Longer Leg}}{\text{Shorter Leg}} = \frac{\text{Longer Leg}}{\text{Shorter Leg}}$$
$$\frac{x}{4.1} = \frac{10.1}{x}$$
$$\frac{x}{4.1} = \frac{10.1}{x}$$
$$x \approx \sqrt{(10.1)(4.1)}$$
$$x \approx \sqrt{41.41}$$
$$\approx 6.4$$

33. Refer to the figure on page 537.



SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg. Let *y* be the shorter segment of the hypotenuse of the bigger right triangle.

$$\frac{7}{4.6} = \frac{4.6}{y}$$
$$7y = 21.16$$
$$y \approx 3.02$$

So, the longer segment of the right triangle is about 3.98 ft.

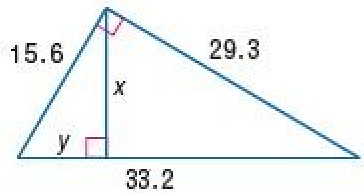
By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

$$x \approx \sqrt{(3.98)(3.02)}$$
$$x \approx \sqrt{12.0196}$$
$$\approx 3.47$$

The length of the segment is about 3.5 ft.

34. Refer to the figure on page 537.

SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg. Let *y* be the shorter segment of the hypotenuse of the bigger right triangle.

$$\frac{\text{Hypotenuse}}{\text{Shorter Leg}} = \frac{\text{Hypotenuse}}{\text{Shorter Leg}}$$

$$\frac{33.2}{15.6} = \frac{15.6}{y}$$

$$33.2y = 15.6 \cdot 15.6$$

$$33.2y = 243.36$$

$$\frac{33.2y}{33.2} = \frac{243.36}{33.2}$$

$$y \approx 7.3$$

So, the longer segment of the right triangle is about 25.9 ft.

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

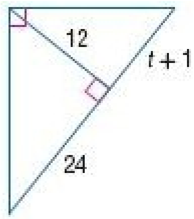
$$x \approx \sqrt{(7.3)(25.9)}$$

$$x \approx \sqrt{189.07}$$

$$\approx 13.8$$

Then *x* is about 13.8 ft.

ALGEBRA Find the value(s) of the variable.

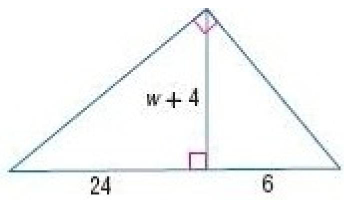
35. 

SOLUTION:

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

$$\frac{\text{Longer Leg}}{\text{Shorter Leg}} = \frac{\text{Longer Leg}}{\text{Shorter Leg}}$$
$$\frac{12}{24} = \frac{t+1}{12}$$
$$12^2 = (24)(t+1)$$
$$144 = 24(t+1)$$
$$144 = 24t + 24$$
$$144 - 24 = 24t + 24 - 24$$
$$120 = 24t$$
$$\frac{120}{24} = \frac{24t}{24}$$
$$5 = t$$

8-1 Geometric Mean

36. 

SOLUTION:

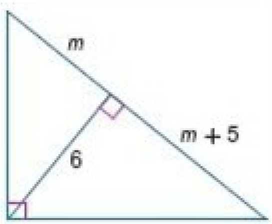
By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

$$(w+4) = \sqrt{(24)(6)}$$
$$(w+4)^2 = 144$$
$$w^2 + 8w + 16 = 144$$
$$w^2 + 8w + 16 - 144 = 144 - 144$$
$$w^2 + 8w - 128 = 0$$

Use the quadratic formula to find the roots of the quadratic equation.

$$w = \frac{-8 \pm \sqrt{(8)^2 - 4(1)(-128)}}{2(1)}$$
$$= \frac{-8 \pm 24}{2}$$
$$x = \frac{-8-24}{2} \quad or \quad x = \frac{-8+24}{2}$$
$$x = \frac{-32}{2} \qquad = \frac{16}{2}$$
$$x = -16 \qquad = 8$$

If $w = -16$, the length of the altitude will be $-16 + 4 = -12$ which is not possible, as a length cannot be negative. Therefore, $w = 8$.

37. 

SOLUTION:

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

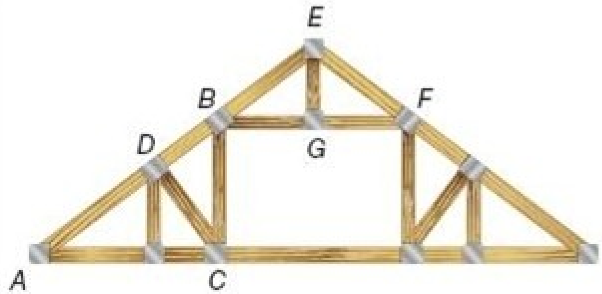
$$\begin{aligned} 6 &= \sqrt{(m)(m+5)} \\ 6^2 &= \left(\sqrt{(m)(m+5)}\right)^2 \\ 36 &= m(m+5) \\ 36 &= m^2 + 5m \\ 36 - 36 &= m^2 + 5m - 36 \\ 0 &= m^2 + 5m - 36 \end{aligned}$$

Use the quadratic formula to find the roots of the quadratic equation.

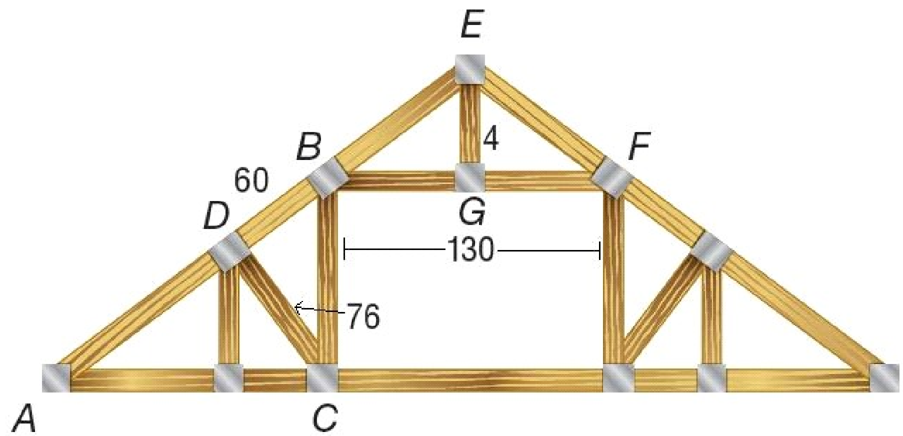
$$\begin{aligned} m &= \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-36)}}{2(1)} \\ &= \frac{-5 \pm 13}{2} \\ m &= \frac{-5-13}{2} \; or \; m = \frac{-5+13}{2} \\ &= \frac{-18}{2} \qquad = \frac{8}{2} \\ &= -9, 4 \end{aligned}$$

Since m is a length, it cannot be negative. Therefore, $m = 4$.

38. **CONSTRUCTION** A room-in-attic truss is a truss design that provides support while leaving area that can be enclosed as living space. In the diagram, $\angle BCA$ and $\angle EGB$ are right angles, $\triangle BEF$ is isosceles, $\overline{CD}$ is an altitude of $\triangle ABC$, and $\overline{EG}$ is an altitude of $\triangle BEF$. If *DB* = 5 feet, *CD* = 6 feet 4 inches, *BF* = 10 feet 10 inches, and *EG* = 4 feet 6 inches, what is *AE*?



SOLUTION:



$\overline{AE} = \overline{AD} + \overline{DB} + \overline{BE}$

First find $\overline{BE}$

Given that $\triangle BEF$ is isosceles, then $\overline{EG}$ bisects $\overline{BF}$. Since $\overline{BF}$ is 10 ft 10 in.or 130 in., then *BG* = *GF* = 65 in. $\triangle BGE$ is a right triangle and by the Pythagorean Theorem,

$BE = \sqrt{EG^2 + BG^2}$.

$$\begin{aligned} BE &= \sqrt{54^2 + 65^2} \\ &= \sqrt{7141} \\ &\approx 84.5 \end{aligned}$$

Next find *AD*. Let *AD* = *x*.

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

$$76 = \sqrt{(x)(60)}$$

$$76^2 = \left(\sqrt{(x)(60)}\right)^2$$

$$5776 = 60x$$

$$96.3 \approx x$$

So, the total length *AE* is about (96.3 + 60 + 84.5) in. = 240.8 in. or 20.07 ft.

PROOF Write a proof for each theorem.

39. Theorem 8.1

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given a right angle and an altitude of a triangle. Use the properties that you have learned about congruent segments, altitudes, right triangles, and equivalent expressions in algebra to walk through the proof.

Given: $\angle PQR$ is a right angle. $\overline{QS}$ is an altitude of ΔPQR.

Prove:

$\Delta PSQ \sim \Delta PQR$

$\Delta PQR \sim \Delta QSR$

$\Delta PSQ \sim \Delta QSR$

Proof:

Statements (Reasons)

1. $\angle PQR$ is a right angle. $\overline{QS}$ is an altitude of ΔPQR. (Given)
2. $\overline{QS} \perp \overline{RP}$ (Definition of altitude)
3. $\angle 1$ and $\angle 2$ are right angles. (Definition of perpendicular lines)
4. $\angle 1 \cong \angle PQR$; $\angle 2 \cong \angle PQR$ (All right angles are congruent.)
5. $\angle P \cong \angle P$; $\angle R \cong \angle R$ (Congruence of angles is reflexive.)
6. $\Delta PSQ \sim \Delta PQR$; $\Delta PQR \sim \Delta QSR$ AA (Similarity Statements 4 and 5)
7. $\Delta PSQ \sim \Delta QSR$ (Similarity of triangles is transitive.)

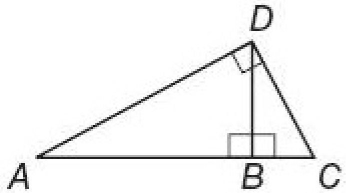
8-1 Geometric Mean

40. Theorem 8.2

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given a right triangle and an altitude. Use the properties that you have learned about right triangles, altitudes, congruent segment,s and equivalent expressions in algebra to walk through the proof.

Given: $\triangle ADC$ is a right triangle. $\overline{DB}$ is an altitude of $\triangle ADC$.



Prove: $\frac{AB}{DB} = \frac{DB}{CB}$

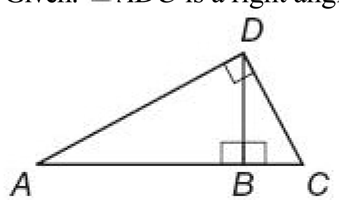
Proof: It is given that $\triangle ADC$ is a right triangle and $\overline{DB}$ is an altitude of $\triangle ADC$. $\angle ADC$ is a right angle by the definition of a right triangle. Therefore, $\triangle ADB \sim \triangle DCB$, because if the altitude is drawn from the vertex of the right angle to the hypotenuse of a right triangle, then the two triangles formed are similar to the given triangle and to each other. So $\frac{AB}{DB} = \frac{DB}{CB}$ by definition of similar triangles.

41. Theorem 8.3

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given a right triangle and an altitude. Use the properties that you have learned about congruent segments, right triangles, altitudes, and equivalent expressions in algebra to walk through the proof.

Given: $\angle ADC$ is a right angle. $\overline{DB}$ is an altitude of $\triangle ADC$.



Prove: $\frac{AB}{AD} = \frac{AD}{AC}; \frac{BC}{DC} = \frac{DC}{AC}$

Proof:

Statements (Reasons)

1. $\angle ADC$ is a right angle. $\overline{DB}$ is an altitude of $\triangle ADC$ (Given)
2. $\triangle ADC$ is a right triangle. (Definition of right triangle)
3. $\triangle ABD \sim \triangle ADC$; $\triangle DBC \sim \triangle ADC$ (If the altitude is drawn from the vertex of the right angle to the hypotenuse of a right triangle, then the 2 triangles formed are similar to the given triangle and to each other.)
4. $\frac{AB}{AD} = \frac{AD}{AC}; \frac{BC}{DC} = \frac{DC}{AC}$ (Definition of similar triangles)

42.

SOLUTION:

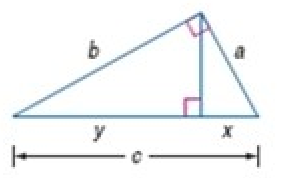
43. **FINANCE** The average rate of return on an investment over two years is the geometric mean of the two annual returns. If an investment returns 12% one year and 7% the next year, what is the average rate of return on this investment over the two-year period?

SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x=\sqrt{ab}$.
Therefore, the geometric mean of 7% and 12% is

$$\sqrt{\left(\frac{7}{100}\right)\left(\frac{12}{100}\right)}=\sqrt{\frac{84}{100}}\approx 9\%.$$

44. **PROOF** Derive the Pythagorean Theorem using the figure at the right and the Geometric Mean (Leg) Theorem.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Use the properties that you have learned about congruent, right triangles, altitudes, and equivalent expressions in algebra to walk through the proof.

Using the Geometric Mean (Leg) Theorem, $a=\sqrt{yc}$ and $b=\sqrt{xc}$. Squaring both values, $a^2 = yc$ and $b^2 = xc$. The sum of the squares is $a^2 + b^2 = yc + xc$. Factoring the c on the right side of the equation, $a^2 + b^2 = c(y + x)$. By the Segment Addition Postulate, $c = y + x$. Substituting, $a^2 + b^2 = c(c)$ or $a^2 + b^2 = c^2$.

8-1 Geometric Mean

Determine whether each statement is always, sometimes, or never true. Explain your reasoning.

45. The geometric mean for consecutive positive integers is the mean of the two numbers.

SOLUTION:

Let x be the first number. Then $x + 1$ is the next. The geometric mean of two consecutive integers is $\sqrt{x(x+1)}$.

The mean of the two number is $\frac{x+(x+1)}{2}$.

Set the two numbers equal.

$$\sqrt{x(x+1)} \stackrel{?}{=} \frac{x+(x+1)}{2}$$

$$\sqrt{x^2+x} \stackrel{?}{=} \frac{2x+1}{2}$$

$$x^2+x \stackrel{?}{=} \frac{(2x+1)^2}{4}$$

$$4x^2+4x \stackrel{?}{=} 4x^2+4x+1$$

$$4x^2-4x^2+4x-4x \stackrel{?}{=} 4x^2-4x^2+4x-4x+1$$

$$0 \neq 1 \qquad =$$

If you set the two expressions equal to each other, the equation has no real solution. Therefore, the statement is *never* true.

46. The geometric mean for two perfect squares is a positive integer.

SOLUTION:

The square root of a perfect square is always a positive integer. Therefore if you multiply two perfect squares, the square root will always be a positive integer. Then since $\sqrt{ab}$ is equal to $\sqrt{a} \cdot \sqrt{b}$, the geometric mean for two perfect squares will always be the product of two positive integers, which is a positive integer. Thus, the statement is *always* true.

Consider the example where a is 16 and b is 25. Then $x = \sqrt{16 \cdot 25} = 20$.

47.

SOLUTION:

48. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate geometric mean.

a. TABULAR Copy and complete the table of five ordered pairs (x, y) such that $\sqrt{xy} = 8$.

x	y	$\sqrt{xy}$
		8
		8
		8
		8
		8

b. GRAPHICAL Graph the ordered pairs from your table in a scatter plot.

c. VERBAL Make a conjecture as to the type of graph that would be formed if you connected the points from your scatter plot. Do you think the graph of any set of ordered pairs that results in the same geometric mean would have the same general shape? Explain your reasoning.

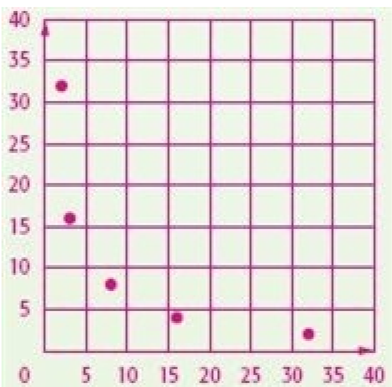
SOLUTION:

a. Find pairs of numbers with product of 64. It is easier to graph these points, if you choose integers.

Sample answer:

x	y	$\sqrt{xy}$
2	32	8
4	16	8
8	8	8
16	4	8
32	2	8

b.

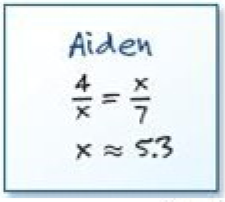


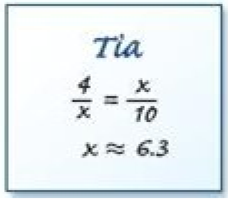
c. What relationship do you notice happening between the x and the y values? Pay close attention to what happens to the y-values, as the x's increase.

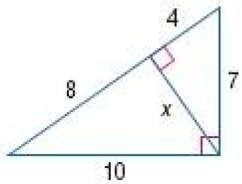
Sample answer:

As x increases, y decreases, and as x decreases, y increases. So, the graph that would be formed will be a hyperbola.

49. **ERROR ANALYSIS** Aiden and Tia are finding the value *x* in the triangle shown. Is either of them correct? Explain your reasoning.







SOLUTION:

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

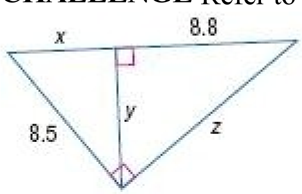
$x = \sqrt{8 \cdot 4}$

$x = \sqrt{32}$

$x \approx 5.7$

Therefore, neither of them are correct.

50. **CHALLENGE** Refer to the figure. Find *x*, *y*, and *z*.



SOLUTION:

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

$\frac{x + 8.8}{8.5} = \frac{8.5}{x}$.

Solve the proportion for *x*.

$$x(x + 8.8) = 8.5 \cdot 8.5$$

$$x^2 + 8.8x = 72.25$$

$x^2 + 8.8x - 72.25 = 0$

Use the quadratic formula to find the roots of the quadratic equation.

$$x = \frac{-8.8 \pm \sqrt{(8.8)^2 - 4(1)(-72.25)}}{2(1)}$$
$$\approx \frac{-8.8 \pm 19.1}{2}$$
$$x \approx \frac{-8.8 - 19.1}{2} \quad or \quad x \approx \frac{-8.8 + 19.1}{2}$$
$$\approx \frac{-27.9}{2} \qquad \approx \frac{10.3}{2}$$
$$\approx -13.95 \qquad \approx 5.15$$

Since x is a length, it cannot be negative. Therefore, x is about 5.2.

By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

$$\frac{x + 8.8}{z} = \frac{z}{8.8}$$

Use the value of x to solve the proportion.

$$z^2 = 8.8(5.2 + 8.8)$$
$$z^2 = 8.8(14)$$
$$z^2 = 123.2$$
$$\sqrt{z^2} = \sqrt{123.2}$$
$$z \approx 11.1$$

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

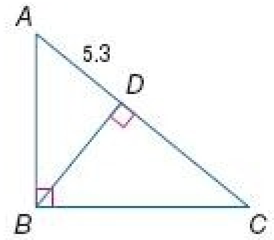
$$y = \sqrt{5.2 \cdot 8.8}$$
$$y = \sqrt{45.76}$$
$$\approx 6.8$$

51. **OPEN ENDED** Find two pairs of whole numbers with a geometric mean that is also a whole number. What condition must be met in order for a pair of numbers to produce a whole-number geometric mean?

SOLUTION:

Sample answer: 9 and 4, 8 and 8; In order for two whole numbers to result in a whole-number geometric mean, their product must be a perfect square. $9 \cdot 4 = 36$ and $8 \cdot 8 = 64$. 36 and 64 are both perfect squares.

52. **REASONING** Refer to the figure. The orthocenter of ΔABC is located 6.4 units from point *D*. Find *BC*.



SOLUTION:

From the figure, *B* is the orthocenter of the triangle *ABC* since *ABC* is a right triangle.
Therefore *BD* = 6.4.

Use the Pythagorean Theorem to find *AB*.

$$\begin{aligned} AB &= \sqrt{BD^2 + AD^2} \\ &= \sqrt{6.4^2 + 5.3^2} \\ &\approx 8.3 \end{aligned}$$

Let *CD* = *x*. Use the Geometric Mean (Altitude) Theorem to find *x*.

$$\begin{aligned} 6.4 &= \sqrt{x(5.3)} \\ 40.96 &= 5.3x \\ x &\approx 7.7 \end{aligned}$$

Use the Pythagorean Theorem to find *BC*.

$$\begin{aligned} BC &= \sqrt{13^2 - 8.3^2} \\ &= \sqrt{AC^2 - AB^2} \\ &\approx 10.0 \end{aligned}$$

53. **WRITING IN MATH** Compare and contrast the arithmetic and geometric means of two numbers. When will the two means be equal? Justify your reasoning.

SOLUTION:

When comparing these two means, consider the commonalities and differences of their formulas, when finding the means of two numbers such as a and b.

Both the arithmetic and the geometric mean calculate a value between two given numbers. The arithmetic mean of two numbers a and b is $\frac{a+b}{2}$, and the geometric mean of two numbers a and b is $\sqrt{ab}$. The two means will be equal when $a = b$.

Justification:

$$\begin{aligned} \frac{a+b}{2} &= \sqrt{ab} \\ \left(\frac{a+b}{2}\right)^2 &= ab \\ (a+b)^2 &= 4ab \\ a^2 + 2ab + b^2 &= 4ab \\ a^2 - 2ab + b^2 &= 0 \\ (a-b)^2 &= 0 \\ a - b &= 0 \\ a &= b \end{aligned}$$

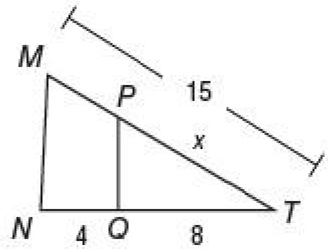
54. What is the geometric mean of 8 and 22 in simplest form?

A $4\sqrt{11}$
B $16\sqrt{11}$
C 15
D 176

SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.
So, the geometric mean of 8 and 22 is
$\sqrt{(8)(22)} = \sqrt{176} = 4\sqrt{11}$.
Therefore, the correct choice is A.

55. **SHORT RESPONSE** If $\overline{MN} \parallel \overline{PQ}$, use a proportion to find the value of x. Show your work.



SOLUTION:

Since $\overline{MN} \parallel \overline{PQ}$, $\overline{PQ}$ divides $\overline{MT}$ and $\overline{NT}$ in the same ratio. So, $\frac{4}{8} = \frac{15-x}{x}$.

Solve the proportion to find the value of x.

$4x = 8(15 - x)$

$4x = 120 - 8x$

$12x = 120$

$x = 10$

56. **ALGEBRA** What are the solutions of the quadratic equation $x^2 - 20 = 8x$?

F 2, 10
G 20, 1
H –1, 20
J –2, 10

SOLUTION:

Write the quadratic equation in standard form.

$x^2 - 8x - 20 = 0$

Use the quadratic formula to find the roots of the quadratic equation.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-20)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 + 80}}{2}$$

$$x = \frac{8 \pm \sqrt{144}}{2}$$

$$x = \frac{8 \pm 12}{2}$$

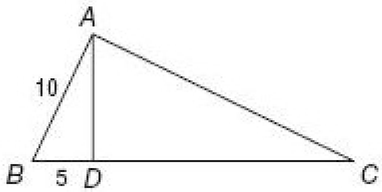
$$x = \frac{8 - 12}{2} \quad \text{or} \quad x = \frac{8 + 12}{2}$$

$$x = \frac{-4}{2} \qquad x = \frac{20}{2}$$

$$x = -2 \qquad x = 10$$

Therefore, the correct choice is J.

57. **SAT/ACT** In the figure, $\overline{AD}$ is perpendicular to $\overline{BC}$, and $\overline{AB}$ is perpendicular to $\overline{AC}$. What is BC?



A $5\sqrt{2}$
B $5\sqrt{3}$
C 20
D 25

SOLUTION:

Since $\overline{AD}$ is perpendicular to $\overline{BC}$, ΔADB is a right triangle. So, by the Pythagorean Theorem, $AD = \sqrt{10^2 - 5^2} = \sqrt{75}.$

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments. So,

$$\sqrt{75} = \sqrt{5 \cdot DC}$$
$$75 = 5 \cdot DC$$
$$\frac{75}{5} = \frac{5DC}{5}$$
$$15 = DC$$

Therefore, $BC = BD + DC = 5 + 15 = 20$.

The correct choice is C.

58. **MAPS** Use the map to estimate how long it would take to drive from Chicago to Springfield if you averaged 65 miles per hour.

SOLUTION:

The scale of the map is 0.5 in. = 100 mi. Use a ruler, the distance between Chicago and Springfield in the map is about 1 inch. Let x be the actual distance between the two cities. Then,

$$\frac{\text{in.}}{\text{mi}} = \frac{\text{in.}}{\text{mi}}$$

$$\frac{0.5}{100} = \frac{1}{x}.$$

Solve the proportion to find the value of x.

$$0.5x = 100$$

$$\frac{0.5x}{0.5} = \frac{100}{0.5}$$

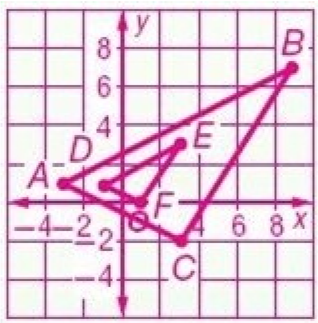
$$x = 200$$

So, the distance between the two cities is about 200 miles. The average speed is 65 miles per hour. Therefore, the time taken to travel from Chicago to Springfield is about $\frac{200}{65} \approx 3$ hr.

Graph the original figure and its dilated image. Then verify that the dilation is a similarity transformation.

59. $A(-3, 1)$, $B(9, 7)$, $C(3, -2)$; $D(-1, 1)$, $E(3, 3)$, $F(1, 0)$

SOLUTION:



Use distance formula to find the lengths of the sides of the two triangles.

$$AB = \sqrt{(9-(-3))^2 + (7-1)^2} = \sqrt{12^2 + 6^2} = \sqrt{180} = \sqrt{36 \cdot 5} = 6\sqrt{5}$$

$$BC = \sqrt{(3-9)^2 + (-2-7)^2} = \sqrt{6^2 + 9^2} = \sqrt{117} = \sqrt{9 \cdot 13} = 3\sqrt{13}$$

$$AC = \sqrt{(3-(-3))^2 + (-2-1)^2} = \sqrt{6^2 + 3^2} = \sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$$

$$DE = \sqrt{(3-(-1))^2 + (3-1)^2} = \sqrt{4^2 + 2^2} = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$$

$$EF = \sqrt{(1-3)^2 + (0-3)^2} = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$DF = \sqrt{(1-(-1))^2 + (0-1)^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$$

Find the ratios of the corresponding sides.

$$\frac{AB}{DE} = \frac{6\sqrt{5}}{2\sqrt{5}} = \frac{6}{2} = 3$$

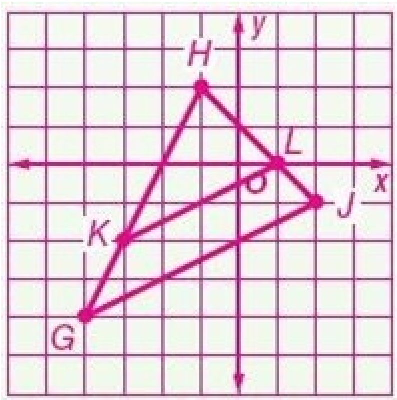
$$\frac{BC}{EF} = \frac{3\sqrt{13}}{\sqrt{13}} = \frac{3}{1} = 3$$

$$\frac{AC}{DF} = \frac{3\sqrt{5}}{\sqrt{5}} = \frac{3}{1} = 3$$

So $\Delta ABC \sim \Delta DEF$ by SSS Similarity.

60. $G(-4, -4)$, $H(-1, 2)$, $J(2, -1)$; $K(-3, -2)$, $L(1, 0)$

SOLUTION:



Here, the vertex H is common to both the triangles. So, $\angle H \cong \angle H$. Find the lengths of the sides adjacent to the vertex H.

$$HG = \sqrt{(-1-(-4))^2 + (2-(-4))^2} = \sqrt{3^2+6^2} = \sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$$

$$HK = \sqrt{(-3-(-1))^2 + (-2-2)^2} = \sqrt{2^2+4^2} = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$$

$$HJ = \sqrt{(2-(-1))^2 + (-1-2)^2} = \sqrt{3^2+3^2} = \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$$

$$HL = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{2^2+2^2} = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$$

Find the ratios of the corresponding sides.

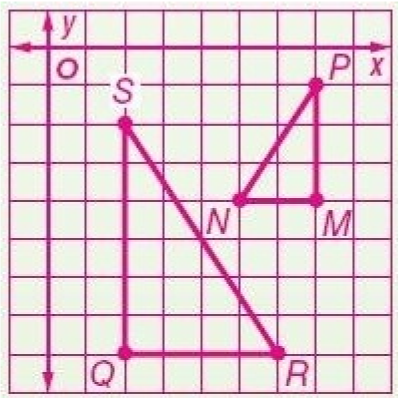
$$\frac{HK}{HG} = \frac{2\sqrt{5}}{3\sqrt{5}} = \frac{2}{3}$$

$$\frac{HL}{HJ} = \frac{2\sqrt{2}}{3\sqrt{2}} = \frac{2}{3}$$

So $\triangle GHJ \sim \triangle KHL$ by SAS Similarity.

61. $M(7, -4)$, $N(5, -4)$, $P(7, -1)$; $Q(2, -8)$, $R(6, -8)$, $S(2, -2)$

SOLUTION:



The angles M and Q are right angles. So, $\angle M \cong \angle Q$. Find the lengths of the sides adjacent to the vertices M and Q.

$$QR = \sqrt{(6-2)^2 + (-8-(-8))^2} = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$$

$$SQ = \sqrt{(2-2)^2 + (-2-(-8))^2} = \sqrt{0^2 + 6^2} = \sqrt{36} = 6$$

$$MN = \sqrt{(5-7)^2 + (-4-(-4))^2} = \sqrt{2^2 + 0^2} = \sqrt{4} = 2$$

$$PM = \sqrt{(7-7)^2 + (-1-(-4))^2} = \sqrt{0^2 + 3^2} = \sqrt{9} = 3$$

Find the ratios of the corresponding sides.

$$\frac{PM}{SQ} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{MN}{QR} = \frac{2}{4} = \frac{1}{2}$$

So $\Delta MNP \sim \Delta QRS$ by SAS Similarity.

The interior angle measure of a regular polygon is given. Identify the polygon.

62. 108

SOLUTION:

Let n = the number of sides in the polygon. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $108n$.

By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$108n = (n-2)180$$
$$108n = 180n - 360$$
$$-72n = -360$$
$$n = 5$$

Therefore, the polygon is a pentagon.

63. 135

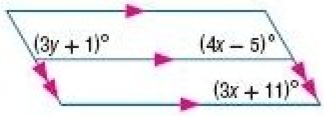
SOLUTION:
Let n = the number of sides in the polygon. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $135n$.

By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$135n = (n-2)180$$
$$135n = 180n - 360$$
$$-45n = -360$$
$$n = 8$$

Therefore, the polygon is an octagon.

Find x and y in each figure.

64. 

SOLUTION:
The angles with the measures $(4x - 5)$ and $(3x + 11)$ are corresponding angles and hence they are equal.

$$4x - 5 = 3x + 11$$
$$x = 16$$

By the Consecutive Interior Angles Theorem, $3y + 1 + 4x - 5 = 180$.

Substitute for x and solve for y.

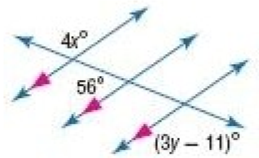
$$3y + 1 + 4(16) - 5 = 180$$
$$3y + 1 = 121$$
$$3y = 120$$
$$y = 40$$

65. 

SOLUTION:
By the Corresponding Angles Postulate, $2x = 68$. So, $x = 34$.

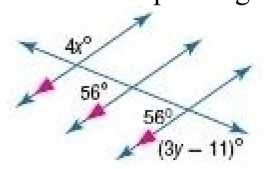
Then, $3x - 15 = 3(34) - 15 = 87$.
By the Corresponding Angles Postulate and triangular sum theorem, $y^2 = 180 - (68 + 87) = 25$

Therefore, $y = \pm 5$.

66. 

SOLUTION:

Use the Corresponding Angle Postulate.



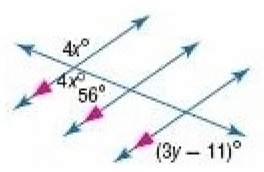
The angles with measures 56 and 3*y* – 11 form a linear pair. So, they are supplementary.

$3y - 11 + 56 = 180$

$3y = 135$

$y = 45$

Now use the Vertical Angle Theorem.

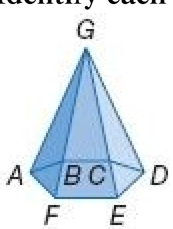


By the Consecutive Interior Angles Theorem, $4x + 56 = 180.$
Solve for *x*.

$4x = 124$

$x = 31$

Identify each solid. Name the bases, faces, edges, and vertices.

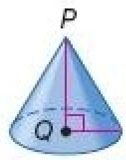
67. 

SOLUTION:

The base of the solid is a hexagon. So, it is a hexagonal pyramid. The base is *ABCDEF*.
The faces, or flat surfaces, are *ABCDEF*, *AGF*, *FGE*, *EGD*, *DGC*, *CGB*, and *BGA*.
The edges are the line segments where faces intersect. The edges, are $\overline{AF}$, $\overline{FE}$, $\overline{ED}$, $\overline{DC}$, $\overline{CB}$, $\overline{BA}$, $\overline{AG}$, $\overline{FG}$, $\overline{EG}$, $\overline{DG}$, $\overline{CG}$, and $\overline{BG}$.
Vertices are points where three or more edges intersect. The vertices are *A*, *B*, *C*, *D*, *E*, *F*, and *G*.

68. 

SOLUTION:
The solid has two circular bases and a curved lateral face. So, it is a cylinder.
The bases are circles with centers P and Q respectively.
A cylinder does not have any faces, edges, or vertices.

69. 

SOLUTION:
The solid has a circular base and a curved lateral face. So, it is a cone. The base is a circle with center Q A cone has faces or edges. The vertex or the cone is is P.

Simplify each expression by rationalizing the denominator.

70. $\frac{2}{\sqrt{2}}$

SOLUTION:

Multiply $\frac{2}{\sqrt{2}}$ by $\frac{\sqrt{2}}{\sqrt{2}}$

Then,

$$\frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}.$$

71. $\frac{16}{\sqrt{3}}$

SOLUTION:

Multiply the numerator and the denominator by $\sqrt{3}$ to rationalize the denominator.

$$\frac{16}{\sqrt{3}} = \frac{16}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{16\sqrt{3}}{3}$$

72. $\frac{\sqrt{6}}{\sqrt{4}}$

SOLUTION:
Simplify the denominator.
$\sqrt{4} = 2$

So, $\frac{\sqrt{6}}{\sqrt{4}} = \frac{\sqrt{6}}{2}$.

73. $\frac{3\sqrt{5}}{\sqrt{11}}$

SOLUTION:

Multiply the numerator and the denominator by $\sqrt{11}$ to rationalize the denominator.

$\frac{3\sqrt{5}}{\sqrt{11}} = \frac{3\sqrt{5}}{\sqrt{11}}\left(\frac{\sqrt{11}}{\sqrt{11}}\right) = \frac{3\sqrt{55}}{11}.$

74. $\frac{21}{\sqrt{3}}$

SOLUTION:

Multiply the numerator and the denominator by $\sqrt{3}$ to rationalize the denominator.

$\frac{21}{\sqrt{3}} = \frac{21}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{21\sqrt{3}}{3} = 7\sqrt{3}.$