State whether each sentence is true or false. If false, replace the underlined term to make a true sentence.

1. A tree diagram uses line segments to display possible outcomes.

SOLUTION:
true

2. A permutation is an arrangement of objects in which order is NOT important.

SOLUTION:
false, combination

3. Determining the arrangement of people around a circular table would require circular permutation.

SOLUTION:
true

4. Tossing a coin and then tossing another coin is an example of dependent events.

SOLUTION:
false, independent

5. Geometric probability involves a geometric measure such as length or area.

SOLUTION:
true

6. 6! = $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, is an example of a factorial.

SOLUTION:
true

7. The set of all possible outcomes is the sample space.

SOLUTION:
true

8. Combining a coin toss and a roll of a die makes a simple event.

SOLUTION:
false, compound

9. Grant flipped a coin 200 times to create a probability tree of the experiment.

SOLUTION:
false, simulation

10. Drawing two socks out of a drawer without replacing them are examples of mutually exclusive events.

SOLUTION:
false, dependent events

11. **POPCORN** A movie theater sells small (S), medium (M), and large (L) size popcorn with the choice of no butter (NB), butter (B), and extra butter (EB). Represent the sample space for popcorn orders by making an organized list, a table, and a tree diagram.

SOLUTION:

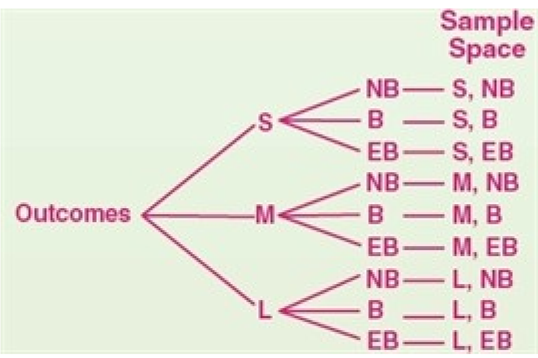
Organized List:
Pair each possible outcome for the size with the possible outcomes for the preference.

S, NB; S, B; S, EB; M, NB; M, B; M, EB; L, NB; L, LB; L, EB

Table:
List the outcomes of the size in the left column and those of the preference in the top row.

Outcomes	No Butter	Butter	Extra Butter
Small	S, NB	S, B	S, EB
Medium	N, NB	M, B	M, EB
Large	L, NB	L, B	L, EB

Tree Diagram:
The top group is all of the outcomes for the size. The second group includes all of the outcomes for the preference. The last group shows the sample space.



12. **SHOES** A pair of men's shoes comes in whole sizes 5 through 13 in navy, brown, or black. How many different pairs could be selected?

SOLUTION:

By the Fundamental Counting Principle the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

The size 5 through 13 can be chosen in 9 different ways and navy, brown, or black can be chosen in 3 different ways. Therefore, the number of different pairs can be chosen in $9 \times 3 = 27$ ways.

13. **DINING** Three boys and three girls go out to eat together. The restaurant only has round tables. Fred does not want any girl next to him and Gena does not want any boy next to her. How many arrangements are possible?

SOLUTION:

Fred does not want any girl next to him and Gena does not want any boy next to her. So, Fred's position is fixed between two boys and Gena's between two girls. Four arrangements can be made by interchanging the positions of the two girls and that of the two boys. So, 4 arrangements are possible.

Let the other boys be Curt and Dwayne. Let the other girls be Jen and Nikki.

C-F-D-J-G-N
D-F-C-J-G-N
C-F-D-N-G-J
D-F-C-N-G-J

14. **DANCE** The dance committee consisted of 10 students. The committee will select three officers at random. What is the probability that Alice, David, and Carlene are selected?

SOLUTION:

3 people can be chosen from 10 people in ${}_{10}C_3$ ways and Alice, David, and Carlene can be chosen in only one way.

$${}_{10}C_3 = \frac{10!}{(10-3)!3!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 3 \cdot 2 \cdot 1} = 120$$

So, the number of possible outcomes is 120 and the number of favorable outcomes is 1. Therefore, the probability is $\frac{1}{120}$.

15. **COMPETITION** From 32 students, 4 are to be randomly chosen for an academic challenge team. In how many ways can this be done?

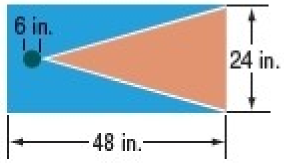
SOLUTION:

The order in which they are chosen does not matter, so it is a combination.

Four students can be chosen from 32 students in ${}_{32}C_4$ ways.

$${}_{32}C_4 = \frac{32!}{(32-4)!4!} = \frac{32 \cdot 31 \cdot 30 \cdot 29 \cdot 28!}{28! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 35,960$$

16. **GAMES** Measurements for a beanbag game are shown. What is the probability of each event?



a. P(hole)
b. P(no hole)

SOLUTION:

a.

$$\text{Area of the hole} = \pi(3)^2 = 9\pi$$

$$\text{Area of the entire region} = 48 \times 24 = 1152$$

$$P(\text{hole}) = \frac{9\pi}{1152} \approx 2.45\%$$

b.

$$P(\text{not hole}) = 1 - P(\text{hole}) \approx 1 - 2.45\% = 1 - \frac{2.45}{100} = \frac{97.55}{100} = 97.55\%$$

17. **POOL** Morgan, Phil, Callie, and Tyreese are sitting on the side of a pool in that order. Morgan is 2 feet from Phil. Phil is 4 feet from Callie. Callie is 3 feet from Tyreese. Oscar joins them.
a. Find the probability that Oscar sits between Morgan and Phil.
b. Find the probability that Oscar sits between Phil and Tyreese.

SOLUTION:

a. The total distance is 2 feet + 4 feet + 3 feet = 9 feet. Morgan is 2 feet from Phil.

$$P(\text{Oscar sits between Morgan and Phil}) = \frac{2}{9}$$

b. The distance between Phil and Tyreese is 4 + 3 = 7 feet.

$$P(\text{Oscar sits between Phil and Tyreese}) = \frac{7}{9}$$

For each of the following, describe how you would use a geometric probability model to design a simulation.

18. **POLO** Max scores 35% of the goals his team earns in each water polo match.

SOLUTION:

Sample answer: Use a spinner that is divided into two sectors, one containing 35% or 126° and the other containing 65% or 234°. Another option would be to use a random number generator. Use 1-100. Let 1-35 represent Max scoring, and let 36-100 represent someone else scoring.

Perform 50 trials and record the results in a frequency table. Use the results to determine the probability of Max scoring in the next match.

19. **BOOKS** According to a survey, people buy 30% of their books in October, November, and December, 22% during January, February, and March, 23% during April, May, and June, and 25% during July, August, and September.

SOLUTION:

Sample answer: Use a spinner that is divided into 4 sectors, 108°, 79.2°, 82.8°, and 90°. Another option would be to use a random number generator. Use 1-100. Let 1-30 represent Oct-Dec, let 31-52 represent Jan-Mar, let 53-75 represent Apr-Jun, and let 76-100 represent Jul-Sep.

Perform 50 trials and record the results in a frequency table. The results can be used to determine the probability of when a particular book will be purchased.

20. **OIL** The United States consumes 17.3 million barrels of oil a day. 63% is used for transportation, 4.9% is used to generate electricity, 7.8% is used for heating and cooking, and 24.3% is used for industrial processes.

SOLUTION:

Sample answer: Use a spinner that is divided into 4 sectors, 226.8°, 87.48°, 28.08°, and 17.64°. With decimal measures, we may want to just approximate the angles. Use 225 for 226.8, 28 for 28.08, etc.

An easier option would be to use a random number generator. Since the percentages have tenths, we need to use 1-1000 in our generator. Let 1-630 represent transportation. Let 631-679 represent electricity. Let 680-757 represent heating. Let 756-1000 represent industrial processes.

Perform 50 trials and record the results in a frequency table. The results can be used to determine the probability for what a certain amount of oil would be used.

21. **MARBLES** A box contains 3 white marbles and 4 black marbles. What is the probability of drawing 2 black marbles and 1 white marble in a row without replacing any marbles?

SOLUTION:

The probability of drawing first black marble of 4 black marbles out of 7 marbles is $\frac{4}{7}$.
The chosen marble is not replaced before the second draw. Therefore, the number of black marbles becomes 3 and the total number of marbles in the box is 6. The probability of drawing second black marble of 3 black marbles out of 6 marbles is $\frac{1}{2}$.

The chosen marble is not replaced before the third draw. Therefore, the total number of marbles in the box is 5. The probability of drawing white marble of 3 white marbles out of 5 marbles is $\frac{3}{5}$.

$$
\begin{aligned}
P(\text{2b and w}) &= P(\text{b}) \times P(\text{b}) \times P(\text{w}) \\
&= \frac{4}{7} \times \frac{1}{2} \times \frac{3}{5} \\
&= \frac{6}{35}
\end{aligned}
$$

22. **CARDS** Two cards are randomly chosen from a standard deck of cards with replacement. What is the probability of successfully drawing, in order, a three and then a queen?

SOLUTION:

The probability of choosing a three of 4 cards out of 52 cards is $\frac{4}{52}$. The chosen card is replaced before the second draw. Therefore, the total number of cards remains the same. The probability of choosing a queen of 4 cards out of 52 cards is $\frac{4}{52}$.

$$
\begin{aligned}
P(\text{3 and q}) &= P(3) \times P(\text{q}) \\
&= \frac{4}{52} \times \frac{4}{52} \\
&= \frac{16}{2704} \\
&= \frac{1}{169}
\end{aligned}
$$

23. **PIZZA** A nationwide survey found that 72% of people in the United States like pizza. If 3 people are randomly selected, what is the probability that all three like pizza?

SOLUTION:

$$
\begin{aligned}
P(\text{all 3 like pizza}) &= (0.72)(0.72)(0.72) \\
&\approx 0.37 \text{ or } 37\%
\end{aligned}
$$

24. **ROLLING DICE** Two dice are rolled. What is the probability that the sum of the numbers is 7 or 11?

SOLUTION:

Since these two events cannot happen at the same time, these are mutually exclusive. The total number of possible outcomes when rolling a pair of dice is 36.

$$\begin{aligned} P(7 \text{ or } 11) &= P(7) + P(11) \\ &= \frac{6}{36} + \frac{2}{36} \\ &= \frac{8}{36} \\ &= \frac{2}{9} \end{aligned}$$

25. **CARDS** A card is drawn from a deck of cards. Find the probability of drawing a 10 or a diamond.

SOLUTION:

Since these two events can happen at the same time, these are not mutually exclusive. Use the rule for two events that are not mutually exclusive.

$$\begin{aligned} P(10 \text{ or d}) &= P(10) + P(\text{d}) - P(10 \text{ and d}) \\ &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\ &= \frac{16}{52} \\ &= \frac{4}{13} \end{aligned}$$

26. **RAFFLE** A bag contains 40 raffle tickets numbered 1 through 40.

a. What is the probability that a ticket chosen is an even number or less than 5?

b. What is the probability that a ticket chosen is greater than 30 or less than 10?

SOLUTION:

a. Since these two events can happen at the same time, these are not mutually exclusive. Use the rule for two events that are not mutually exclusive.

$$\begin{aligned} P(\text{e or } < 5) &= P(\text{e}) + P(< 5) - P(\text{e and } < 5) \\ &= \frac{20}{40} + \frac{4}{40} - \frac{2}{40} \\ &= \frac{22}{40} \\ &= \frac{11}{20} \end{aligned}$$

b. Since these two events can not happen at the same time, these are mutually exclusive. Use the rule for two events that are not mutually exclusive.

$$\begin{aligned} P(> 30 \text{ or } < 10) &= P(> 30) + P(< 10) \\ &= \frac{10}{40} + \frac{9}{40} \\ &= \frac{19}{40} \end{aligned}$$